



Virtual University of Pakistan
FEDERAL GOVERNMENT UNIVERSITY

COURSE HANDOUTS

Statics

MTH304

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

IN THE NAME OF ALLAH
THE MOST BENEFICENT
THE MOST MERCIFUL

اِسْلَامُ عَلَیْكَ يَا سُوْلَ اللَّهِ
وَعَلَىٰ اَکْصَحِّکَ مَا حَدَّثَ اللَّهُ

Lecture No 1

Mechanics

Mechanics is the branch of physical Sciences which studies the action of force on bodies. Under the action of these forces, bodies are in state of rest or of motion relative to some frame of reference.

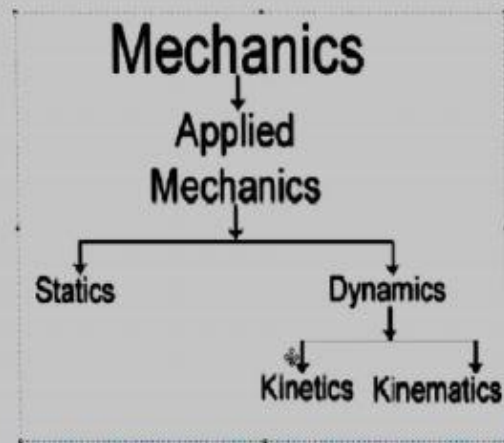
Mechanics is further divided into two areas

Statics

Statics is that branch of mechanics which deals with forces acting on a body which is at rest.

Dynamics

Dynamics is that branch of mechanics which accelerated motion. |



Some basic definitions

Body

Matter which supplies limited portion of space is called a body.

Mass

The quantity of a matter in a body is called its mass.

Particle

A body of negligibly small dimension is termed a particle.

Rigid body

A body is said to be a rigid body if the distance between any two particles of the body remains unchanged so that the body does not expand or contract or change its shape in any way.

Rest and motion

A body is said to be in motion if, in the course of time, its position changes with respect to its surroundings, otherwise the body is said to be in rest. Rest and motion are relative concepts and are meaningful only when referred to surrounding bodies.

Inertial and Non-Inertial Reference Frames

- Recall: Newton's first law of motion (law of inertia):

An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

Inertial and Non-Inertial Reference Frames

- We've already talked about frames of reference in this course when we talked about relative velocity. You know that a person sitting on a bus watching a ball roll across the ground will have a very different measurement of the ball's velocity than a person standing outside the bus would measure the ball's velocity.

Inertial and Non-Inertial Reference Frames

- The person inside the bus and the person outside the bus are measuring from different **frames of reference**.

Inertial and Non-Inertial Reference Frames

- The laws of physics seem to momentarily break down for you sitting on the bus. In reality, what has happened is that your frame of reference has been **compromised**.

Inertial and Non-Inertial Reference Frames

- An **inertial frame of reference** is a frame of reference in which the law of inertia and other physics laws **are valid**. Any frame moving at a constant velocity relative to another frame is also an inertial frame of reference.

Inertial and Non-Inertial Reference Frames

- When the breaks are applied to the bus, the bus undergoes a negative acceleration. At this moment, it becomes a non-inertial frame of reference.
- A **non-inertial frame** of reference is a reference frame in which **the law of inertia does not hold**.

Inertial and Non-Inertial Reference Frames

- An **inertial frame of reference** is a frame of reference in which the law of inertia and other physics laws **are valid**. Any frame moving at a constant velocity relative to another frame is also an inertial frame of reference.

Inertial and Non-Inertial Reference Frames

- Although the ball accelerates toward the front of the bus, there is **no net force** causing the acceleration.
- But if you are sitting on the bus, you observe the ball accelerating forward. That would imply to you as you sit on the bus that there is a **net force forward** on the ball.
- The reason there appears to be a net force on the ball is that you are observing the motion of the ball in the non-inertial reference frame.

Inertial and Non-Inertial Reference Frames

- If you were observing the motion from the road (which is an inertial frame of reference) the ball just continues to move forward at the speed it was already going, and its motion is easily explained by the law of inertia.
- To an observer in the inertial frame of reference (the ground) the *bus* experiences a net force causing it to decelerate. The *ball* just continues its forward velocity with no net force.

What is a Force?

A force is a push or pull acting on an object that changes the motion of the object.

Types of Forces

- **Contact Force** – Forces that act through direct contact between two objects
 - Applied Forces, Friction
- **Long Range Forces** – Forces that can act over distances
 - Gravity, Electromagnetic Force (EMF)

Newton's Second Law of Motion

- Newton discovered the idea of a Force
- He found the Force is proportional to the Acceleration of an object (more Force = more Acceleration)
- He found the Force is proportional to the mass of the object (more mass = more force needed).

Newton's Second Law of Motion

- Moving objects accelerate when an unbalanced force (F) acts on them. The stronger the force, the greater the acceleration (a). Also, the greater the mass (m) the greater the force required to change the motion.
- Force = mass x acceleration
 - $F = ma$

Newton's Second Law of Motion

- Force is a Vector Quantity and therefore has magnitude and direction.
- The direction of the force is the same as the direction of the acceleration.

S.I. Unit For Force

- The Unit for Force is a Newton (N)
 - $1\text{N} = 1\text{kg m/s}^2$
 - A Newton (N) is defined as the amount of force required to accelerate 1 kg of mass at a rate of 1 m/s^2 .

Net Force

- When Multiple forces are acting on an object. The Net Force is the amount of force that is left after adding all the forces on the object.
- Net force (F_{NET}) = Resultant Force (F_{R})

Balanced Forces

Balanced Forces are forces that are equal and opposite so that they cancel out.

10 N East

10 N West



$$\text{Net Force} = +10 \text{ N} + -10 \text{ N} = 0$$

Unbalanced Forces

Unbalanced Forces are forces that when added do not cancel out and cause a change in the motion of the object.

30 N East

10 N West



$$F_{\text{net}} = +30 \text{ N} + -10 \text{ N} = +20 \text{ N East}$$

Inertia

Inertia measures the tendency of an object to resist changes in motion.

- Galileo came up with the idea of inertia
- Objects do not want their motion to change
- Mass measures how much inertia an object has (More mass = More inertia)

Newton's First Law of Motion (Law of Inertia)

- If no unbalanced forces act on a moving object, then the object will continue to move with a constant velocity (constant speed in a straight line). If an object is at rest it will stay at rest.
- Newton took his concept of forces and combined it with Galileo's idea of inertia

Equilibrium

- First Condition for Equilibrium
 - If the Net Force acting on the object is zero
$$F_{\text{NET}} = 0 \quad a = 0$$
 - The object is either stationary ($v = 0$) or traveling with a constant velocity ($v = \text{constant}$)

Mass and Weight

Mass is amount of matter that an object possess. Mass does not change with location.

Weight is the gravitational force that a large body (such as a planet) exerts on another object.

Weight

- Weight is a Force! It is measured in Newtons.
- Weight = Mass x Acceleration due to Gravity (Newton's 2nd law)
- $W = mg$
- Weight does change with location! ("g" will change with location)

Point of Application of a Force

The point of the body where the force acts is called the point of application.

Line of action of a Force

The line passing through the point of application in the direction of the force is called its line of action.

Force System

A set of forces acting on a particle or rigid body is called a force system or system of forces.

Equivalent force system

Two force system are said to be equivalent if one sysem can be replaced by the other without disturbing the instataneous state of rest or motion of a body.

Point of Application of a Force

The point of the body where the force acts is called the point of application.

Line of action of a Force

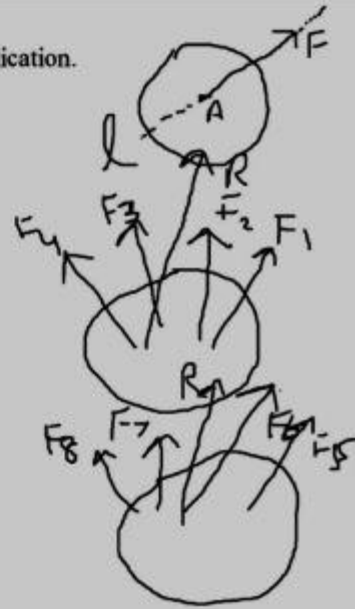
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**Law of Superposition**

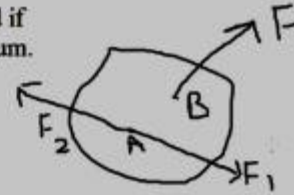
The action of a given system of forces on a body remains unchanged if we add to or subtract from them another system of forces in equilibrium.

Principle of Transmissibility of Forces

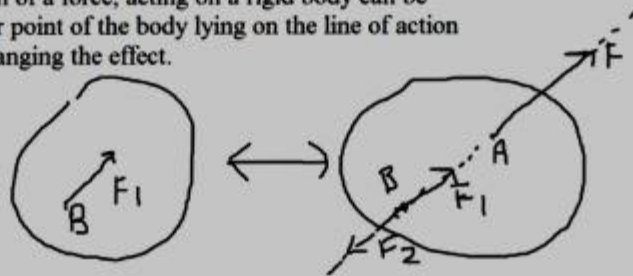
The point of application of a force, acting on a rigid body can be transferred to any other point of the body lying on the line of action of the force without changing the effect.

Law of Superposition

The action of a given system of forces on a body remains unchanged if we add to or subtract from them another system of forces in equilibrium.

**Principle of Transmissibility of Forces**

The point of application of a force, acting on a rigid body can be transferred to any other point of the body lying on the line of action of the force without changing the effect.

**VECTOR**

The term vector is used by scientists to indicate a quantity (such as displacement or velocity or force) that has both magnitude and direction.

REPRESENTING A VECTOR

A vector is often represented by an arrow or a directed line segment.

- The length of the arrow represents the magnitude of the vector.
- The arrow points in the direction of the vector.

DENOTING A VECTOR

We denote a vector by either:

- Printing a letter in boldface ($\mathbf{v}$)
- Putting an arrow above the letter ($\vec{v}$)

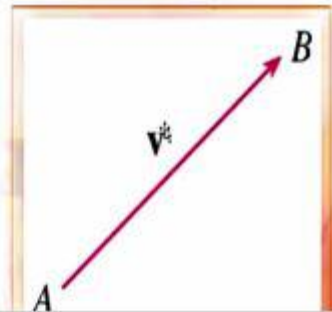
VECTORS

For instance, suppose a particle moves along a line segment from point A to point B .

VECTORS

The corresponding displacement vector $\mathbf{v}$ has initial point A (the tail) and terminal point B (the tip).

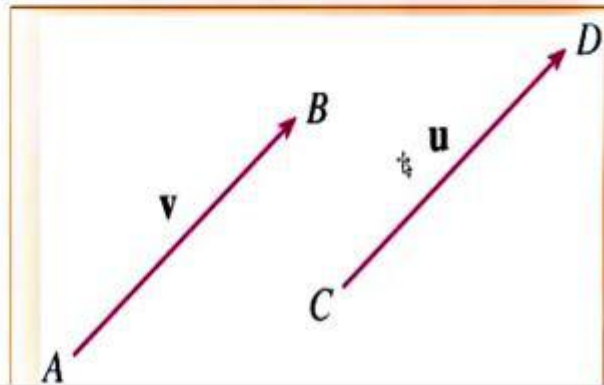
- We indicate this by writing $\mathbf{v} = \overrightarrow{AB}$.



VECTORS

Notice that the vector $\mathbf{u} = \overrightarrow{CD}$ has the same length and the same direction as $\mathbf{v}$ even though it is in a different position.

- We say $\mathbf{u}$ and $\mathbf{v}$ are equivalent (or equal) and write $\mathbf{u} = \mathbf{v}$.



ZERO VECTOR

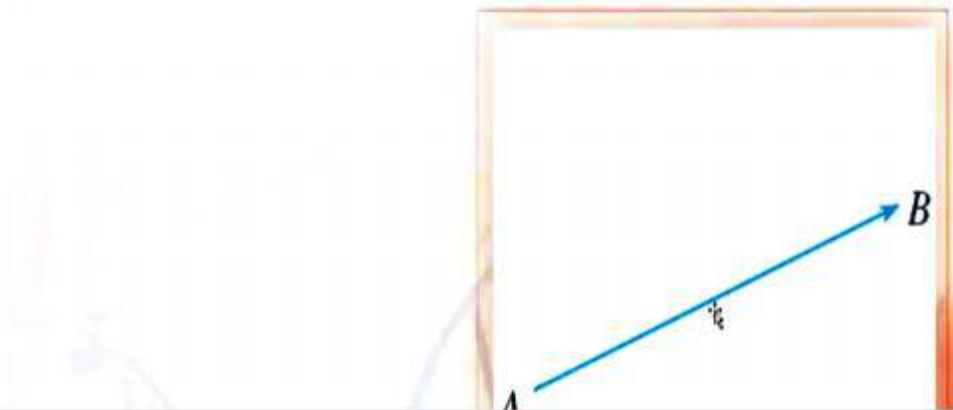
The zero vector, denoted by $\mathbf{0}$, has length 0.

- It is the only vector with no specific direction.

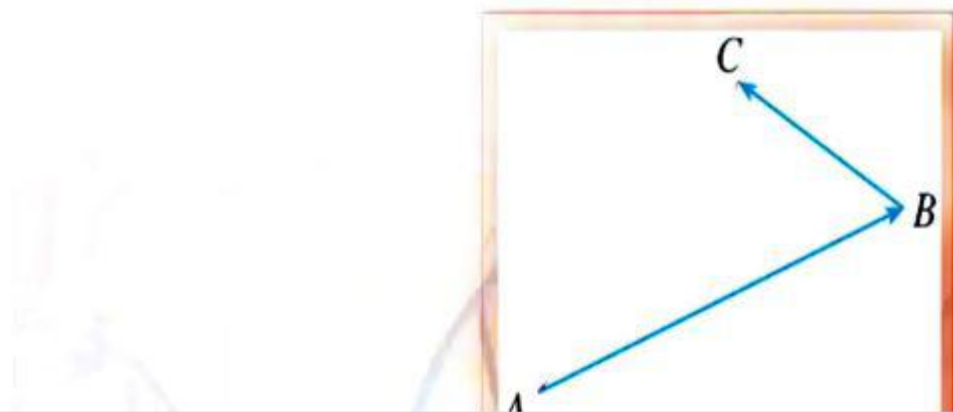
COMBINING VECTORS

Suppose a particle moves from A to B .

So, its displacement vector is $\overrightarrow{AB}$.

**COMBINING VECTORS**

Then, the particle changes direction,
and moves from B to C —with displacement
vector $\overrightarrow{BC}$.

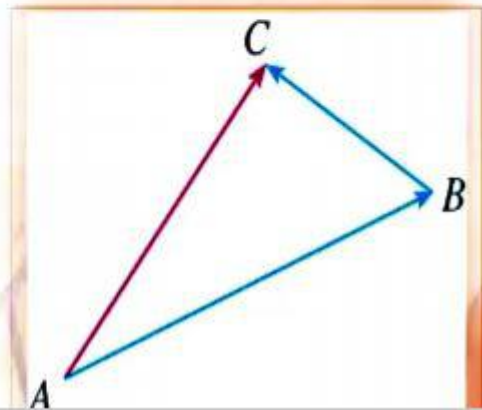


COMBINING VECTORS

The resulting displacement vector $\overrightarrow{AC}$ is called the sum of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We write:

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

**ADDING VECTORS**

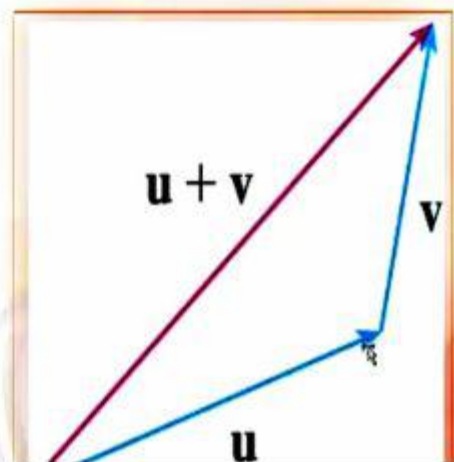
In general, if we start with vectors u and v , we first move v so that its tail coincides with the tip of u and define the sum of u and v as follows.

VECTOR ADDITION—DEFINITION

If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the sum $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.

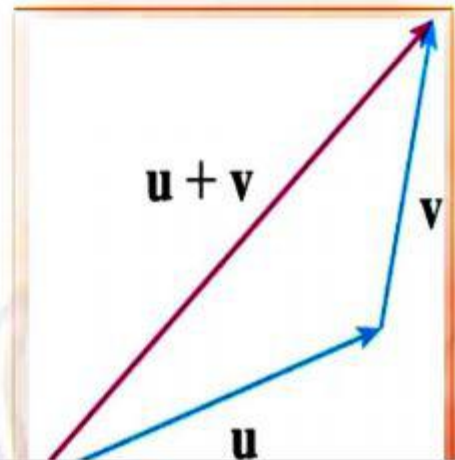
VECTOR ADDITION

The definition of vector addition is illustrated here.

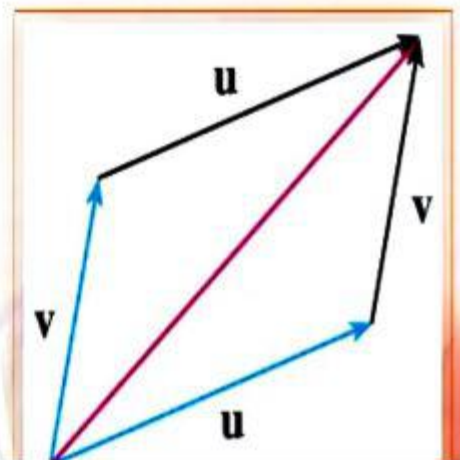


TRIANGLE LAW

You can see why this definition is sometimes called the Triangle Law.

**VECTOR ADDITION**

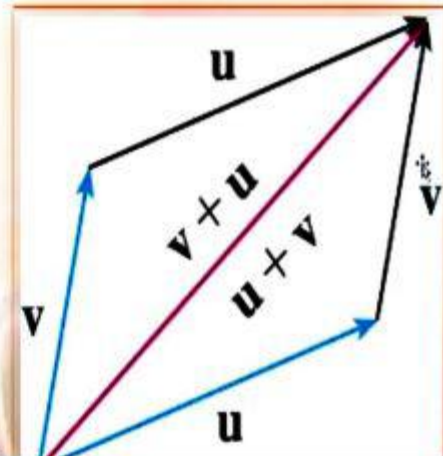
Here, we start with the same vectors u and v as earlier and draw another copy of v with the same initial point as u .



VECTOR ADDITION

Completing the parallelogram,
we see that:

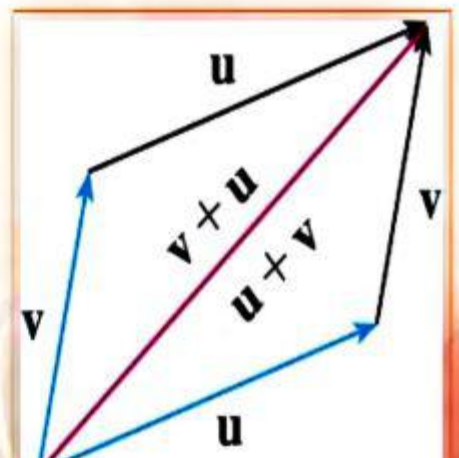
$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$



VECTOR ADDITION

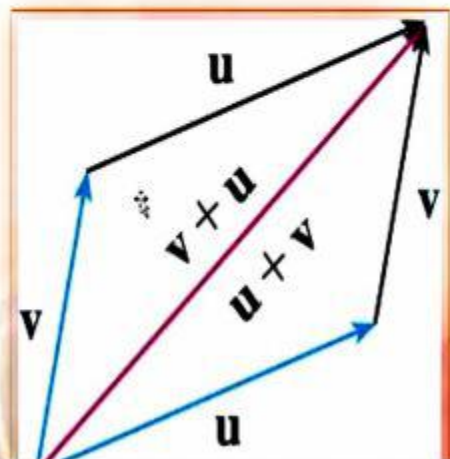
This also gives another way to construct
the sum:

- If we place $\mathbf{u}$ and $\mathbf{v}$ so they start at the same point, then $\mathbf{u} + \mathbf{v}$ lies along the diagonal of the parallelogram with $\mathbf{u}$ and $\mathbf{v}$ as sides.

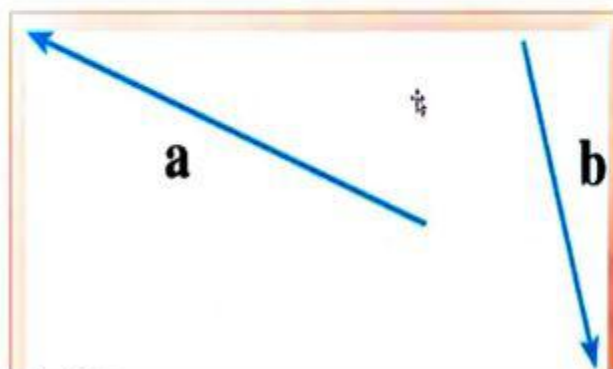


PARALLELOGRAM LAW

This is called the Parallelogram Law.

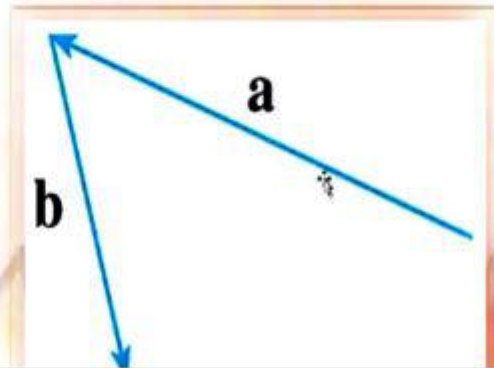
**VECTOR ADDITION****Example 1**

Draw the sum of the vectors a and b shown here.

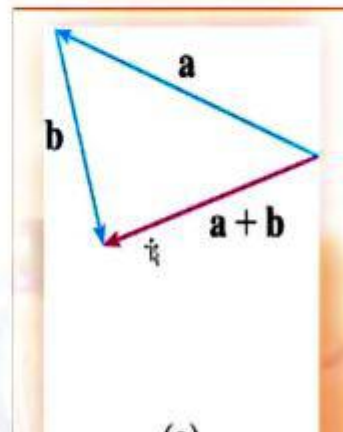


VECTOR ADDITION**Example 1**

First, we translate $\mathbf{b}$ and place its tail at the tip of $\mathbf{a}$ —being careful to draw a copy of $\mathbf{b}$ that has the same length and direction.

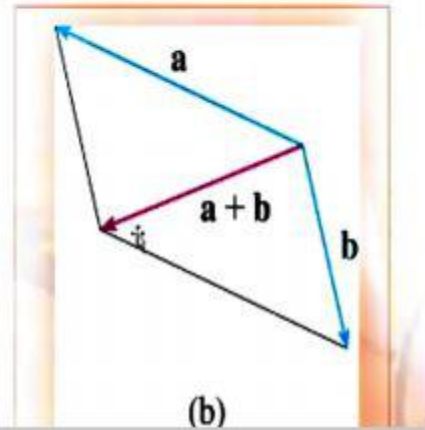
**VECTOR ADDITION****Example 1**

Then, we draw the vector $\mathbf{a} + \mathbf{b}$ starting at the initial point of $\mathbf{a}$ and ending at the terminal point of the copy of $\mathbf{b}$.



VECTOR ADDITION**Example 1**

Alternatively, we could place $\mathbf{b}$ so it starts where $\mathbf{a}$ starts and construct $\mathbf{a} + \mathbf{b}$ by the Parallelogram Law.

**MULTIPLYING VECTORS**

It is possible to multiply a vector by a real number c .

SCALAR

In this context, we call the real number c a scalar—to distinguish it from a vector.

MULTIPLYING SCALARS

For instance, we want $2\mathbf{v}$ to be the same vector as $\mathbf{v} + \mathbf{v}$, which has the same direction as $\mathbf{v}$ but is twice as long.

In general, we multiply a vector by a scalar as follows.

If c is a scalar and $\mathbf{v}$ is a vector, the scalar multiple $c\mathbf{v}$ is:

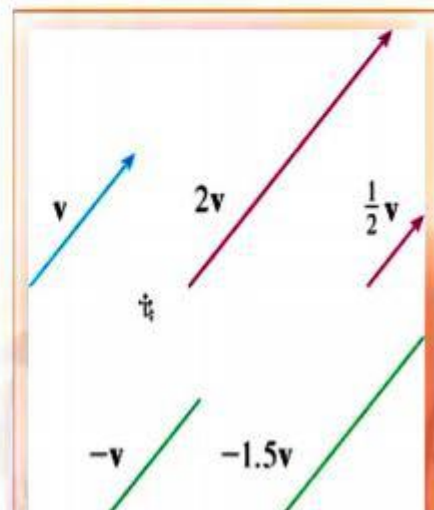
The vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose direction is the same as $\mathbf{v}$ if $c > 0$ and is opposite to $\mathbf{v}$ if $c < 0$.

- If $c = 0$ or $\mathbf{v} = \mathbf{0}$, then $c\mathbf{v} = \mathbf{0}$.

SCALAR MULTIPLICATION

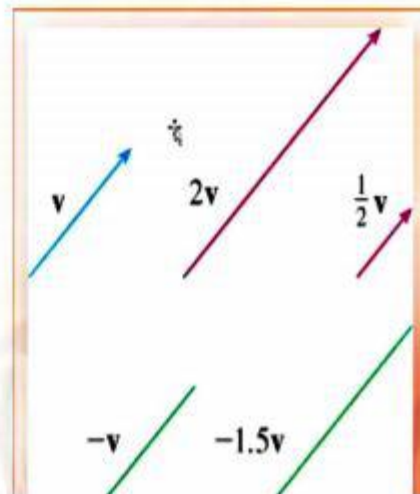
The definition is illustrated here.

- We see that real numbers work like scaling factors here.
- That's why we call them scalars.



SCALAR MULTIPLICATION

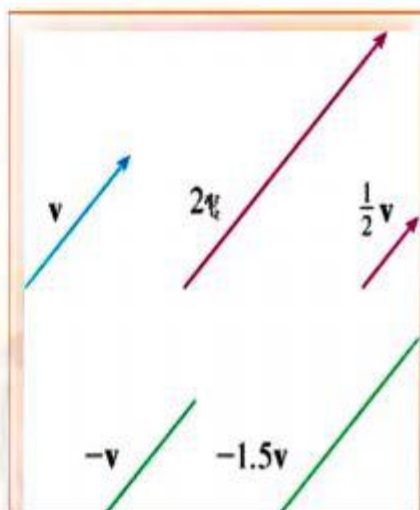
Notice that two nonzero vectors are parallel if they are scalar multiples of one another.



SCALAR MULTIPLICATION

In particular, the vector $-v = (-1)v$ has the same length as v but points in the opposite direction.

- We call it the negative of v .



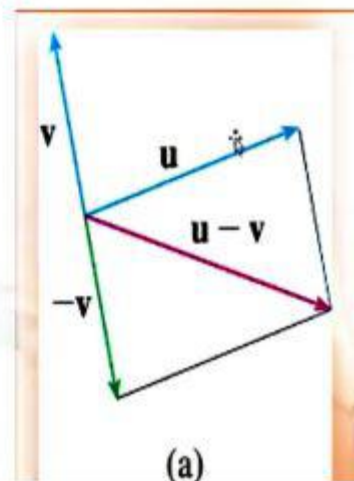
SUBTRACTING VECTORS

By the difference $\mathbf{u} - \mathbf{v}$ of two vectors, we mean:

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$

SUBTRACTING VECTORS

So, we can construct $\mathbf{u} - \mathbf{v}$ by first drawing the negative of $\mathbf{v}$, $-\mathbf{v}$, and then adding it to $\mathbf{u}$ by the Parallelogram Law.

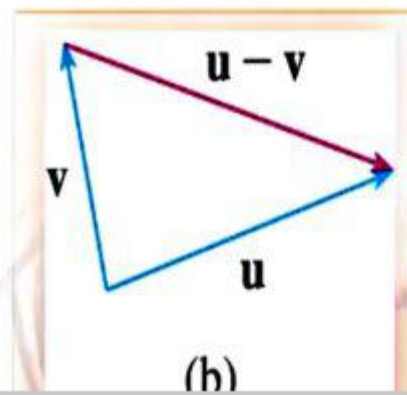


SUBTRACTING VECTORS

Alternatively, since $\mathbf{v} + (\mathbf{u} - \mathbf{v}) = \mathbf{u}$,
the vector $\mathbf{u} - \mathbf{v}$, when added to $\mathbf{v}$,
gives $\mathbf{u}$.

SUBTRACTING VECTORS

So, we could construct $\mathbf{u}$ by means of
the Triangle Law.



Example

Two forces of magnitude 8N and 10 N are acting on a particle as shown in the figure. Find the resultant of these forces by applying parallelogram law.

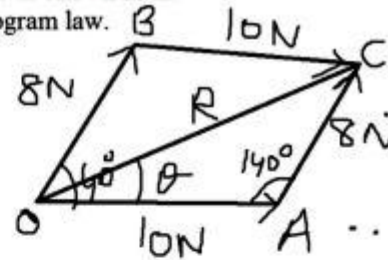
Find also the angle made by the resultant with 10N force.

Sol

$\triangle OAC$

Law of Cosines

$$\begin{aligned} R^2 &= 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 140^\circ \\ &= 64 + 100 - 160 \cos 140^\circ \\ &= 164 + 122.57 \\ &= 286.57 \end{aligned}$$



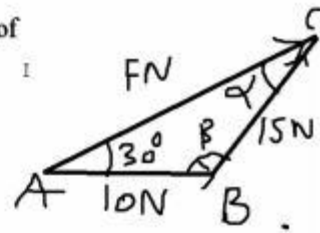
$$R = 17 \text{ N}$$

Law of Sines

$$\begin{aligned} \frac{R}{\sin 140^\circ} &= \frac{8}{\sin \theta} \\ \frac{17}{\sin 140^\circ} &= \frac{8}{\sin \theta} \\ \sin \theta &= \frac{8}{17} \times \sin 140^\circ \\ &= 0.3 \\ \theta &= \sin^{-1}(0.3) \\ \theta &= 17.6^\circ \end{aligned}$$

Example

Two forces of magnitude 10N and 15N are the resolved parts of force of magnitude FN as shown in the figure. Find the magnitude of Force F.



Sol;

Using law of sines

$$\frac{15}{\sin 30^\circ} = \frac{10}{\sin \alpha}$$

$$\sin \alpha = \frac{10}{15} \times \sin 30^\circ = \frac{1}{3} \Rightarrow \alpha = \sin^{-1}\left(\frac{1}{3}\right) = 19.5^\circ$$

$$\beta = 180^\circ - 30^\circ - 19.5^\circ = 130.5^\circ$$

Now using law of cosines

$$F^2 = 10^2 + 15^2 - 2 \times 10 \times 15 \cos 130.5^\circ$$

$$= 130.5^\circ$$

Now using law of cosines

$$F^2 = 10^2 + 15^2 - 2 \times 10 \times 15 \cos 130.5^\circ$$

$$= 325 - 300 \cos 130.5^\circ$$

$$= 325 + 194.83$$

$$= 519.83$$

$$F = \sqrt{519.83}$$

$$\Rightarrow F = 22.8 \text{ N}$$

Example

Two forces of magnitude 100N and 80N are acting on a particle as shown in the fig.

(i) Find the magnitude of the resultant by applying parallelogram law.

(ii) Find the angle that the resultant makes with vertical.

Sol Using Law of cosines

$$R^2 = 100^2 + 80^2 - 2 \times 100 \times 80 \times \cos 70^\circ$$

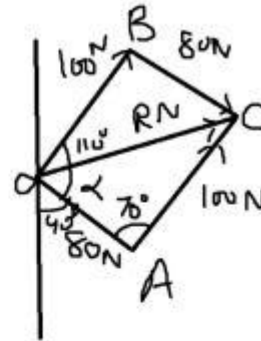
$$R = \sqrt{161400 - 16000 \cos 70^\circ}$$

$$= 104.54$$

Using Law of Sines

$$\frac{\sin d}{100} = \frac{\sin 70^\circ}{104.54}$$

$$\sin d = \frac{100 \sin 70^\circ}{104.54}$$



(ii) Find the angle that the resultant makes with vertical.

Sol Using Law of cosines

$$R^2 = 100^2 + 80^2 - 2 \times 100 \times 80 \times \cos 70^\circ$$

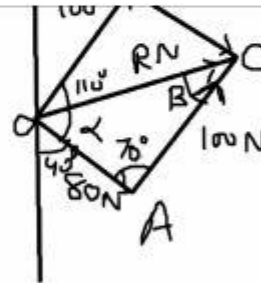
$$R = \sqrt{161400 - 16000 \cos 70^\circ}$$

$$= 104.54 \text{ N}$$

Using Law of Sines

$$\frac{\sin d}{100} = \frac{\sin 70^\circ}{104.54}$$

$$\sin d = \frac{100 \sin 70^\circ}{104.54}$$



$$R = \sqrt{161400 - 16000 \cos 70^\circ}$$

$$= 104.54 \text{ N}$$

Using Law of Sines

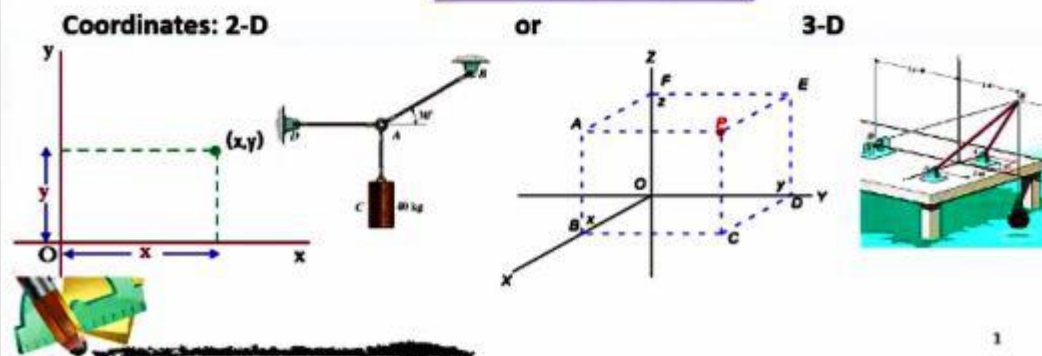
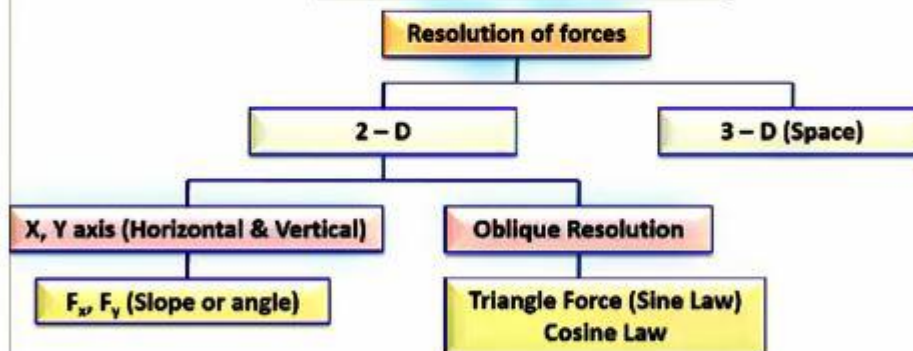
$$\frac{\sin \alpha}{160} = \frac{\sin 70^\circ}{104.54}$$

$$\sin \alpha = \frac{160 \sin 70^\circ}{104.54}$$

$$\alpha = 64^\circ$$

$$\text{Angle with vertical} = 64^\circ + 4^\circ = 104^\circ$$

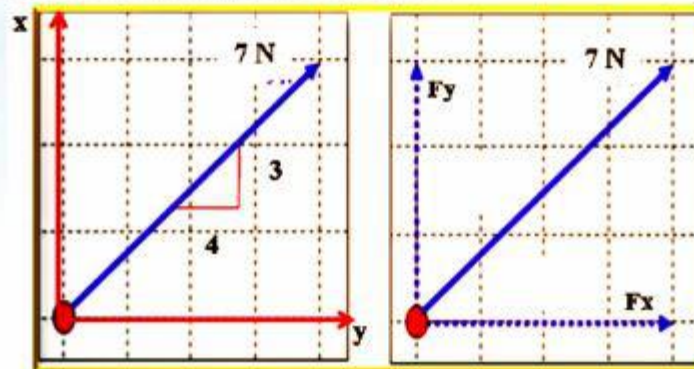
Resolution of Forces



Resolution of Forces

❖ The process of replacing a force system by its components is called **Resolution**.

❖ To resolve a force into x-y components means to express the force as the sum of two forces, one in the x-direction (the x-component) and one in the y-direction (the y-component). When resolving a force into x-y components, we must have information on the direction of the force and the magnitude of the force.



$$F_x = 7 \times \left(\frac{4}{5}\right)$$

$$F_y = 7 \times \left(\frac{3}{5}\right)$$

$$F_x = 7 \times \cos 36.7^\circ$$

$$F_y = 7 \times \sin 36.7^\circ$$

2

Resolution of Forces

➤ When the **angle of the force relative to the x- or y-axes** is known, we can use trigonometry to find the components. Let (θ) be the angle that the force makes with the positive x-axis. Using trigonometry, we find the components (F_x and F_y) as follows:

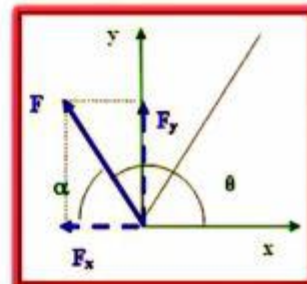
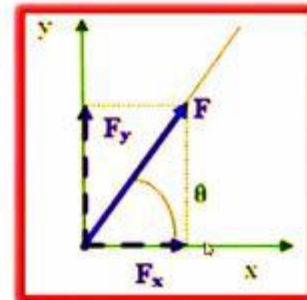
$$\sin \theta = \frac{F_y}{F} \Rightarrow F_y = F \cdot \sin \theta$$

$$\cos \theta = \frac{F_x}{F} \Rightarrow F_x = F \cdot \cos \theta$$

➤ It is usually easiest to find the magnitudes of the components from the acute angle of the triangle defined by the force and the axes. Note that the components may have positive or negative signs. You need to recognize the signs of the components so they agree with their senses.

$$\sin \theta = \frac{F_y}{F} = \sin(180 - \alpha) \Rightarrow F_y = F \cdot \sin \alpha$$

$$\cos \theta = \frac{F_x}{F} = \cos(180 - \alpha) \Rightarrow F_x = -F \cdot \cos \alpha$$



3

Resolution of Forces

Oblique Components (Oblique Forces):

- ❖ If non-rectangular components of a force are needed, several methods are available for determining them.
- ❖ The components of the force (F) shown as (OA) and (OB) can be determined graphically by drawing the **parallelogram** to any convenient scale. The magnitudes of the components can be determined algebraically from the law of sines & cosines [With reference to triangle (ABC) with sides of (a, b, c), the sine rule states] which states:

Parallelogram Laws:

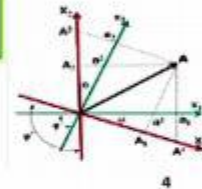
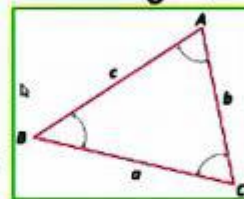
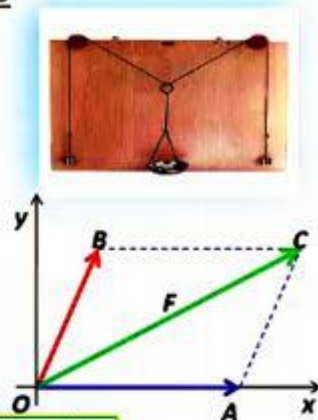
Sine Law:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Law:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Example 1

- ❖ Resolve the (1000 N) force acting on the pipe, into components in the (a) **X** and **Y** directions, and (b) **X'** and **Y'** directions.

Solution:

Part (a): note that the length of the components is scaled along the X and Y axes by first constructing lines from the tip of ($F = 1000 \text{ N}$) parallel to the axes in accordance with the parallelogram law.

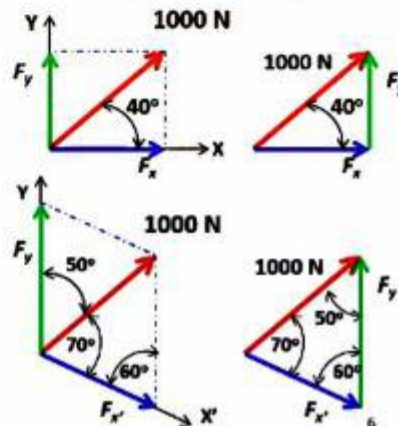
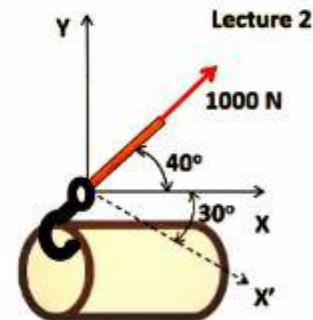
$$F_x = 1000 \cos 40^\circ = 766 \text{ N}$$

$$F_y = 1000 \sin 40^\circ = 643 \text{ N}$$

Part (b): note carefully how the parallelogram is constructed. Applying the law of sines using data mentioned in figure, it will yield:

$$\frac{F_{x'}}{\sin 50^\circ} = \frac{1000}{\sin 60^\circ} \rightarrow F_{x'} = 1000 \cdot \frac{\sin 50^\circ}{\sin 60^\circ} = 884.6 \text{ N}$$

$$\frac{F_y}{\sin 70^\circ} = \frac{1000}{\sin 60^\circ} \rightarrow F_y = 1000 \cdot \frac{\sin 70^\circ}{\sin 60^\circ} = 1085 \text{ N}$$



Example 2

❖ The screw eye is subjected to two forces, F_1 and F_2 . determine the magnitude and direction of the resultant force.

Solution:

Parallelogram Law: The parallelogram law of addition is shown in figure below. The two unknowns are the magnitude of (F_R) and the angle (θ).

Trigonometry: from the figure, the vector triangle is constructed. F_R is determined by using the law of cosines:

$$F_R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115}$$

$$= \sqrt{10000 + 22500 - 30000(-0.4226)} = 212.6 \text{ N}$$

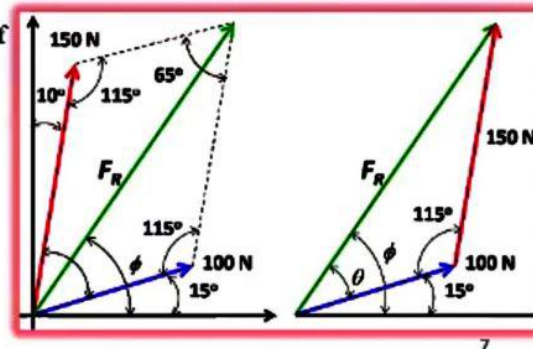
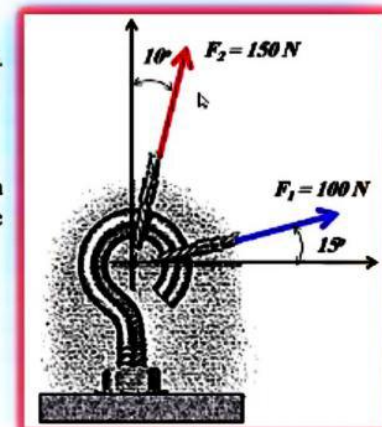
The angle (θ) is determined by applying the law of sines, using the computed value of (F_R):

$$\frac{150}{\sin \theta} = \frac{212.6}{\sin 115} \rightarrow \sin \theta = \frac{150}{212.6} (0.9063)$$

$$\theta = 39.8^\circ$$

Thus, the direction (ϕ) of (F_R), measured from the horizontal is:

$$\phi = 39.8 + 15 = 54.8^\circ$$



Example 2

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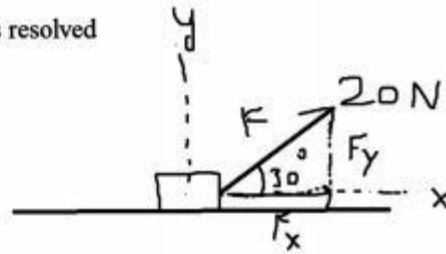
$$\phi = 39.8 + 15 = 54.8^\circ$$

Example

A force of 20N is acting on a block as shown in fig. Find its resolved parts along the coordinate axes.

Sol

$$\begin{aligned}
 F_x &= F \cos \theta \\
 &= 20 \cos 30^\circ \\
 &= 20 \left(\frac{\sqrt{3}}{2} \right) \\
 &= 10\sqrt{3} \text{ N}
 \end{aligned}$$



$$\begin{aligned}
 &= 20 \cos 30^\circ \\
 &= 20 \left(\frac{\sqrt{3}}{2} \right) \\
 &= 10\sqrt{3} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 F_y &= F \sin \theta \\
 &= 20 \sin 30^\circ \\
 &= 20 \left(\frac{1}{2} \right) \\
 &= 10 \text{ N}
 \end{aligned}$$

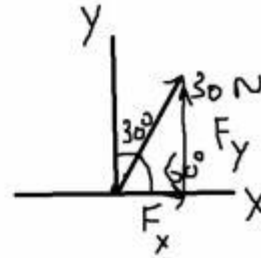
Example

a force of 30N is acting on the particle at an angle of 30 degrees with y-axis. Find the resolved parts along x-axis and y-axis.

Sol

$$\text{Angle with X-axis} = 90^\circ - 30^\circ = 60^\circ$$

$$\begin{aligned} F_x &= F \cos \theta \\ &= 30 \cos 60^\circ \\ &= 30 \left(\frac{1}{2} \right) \\ F_x &= 15 \text{ N} \end{aligned}$$



$$\begin{aligned} F_x &= F \cos \theta \\ &= 30 \cos 60^\circ \\ &= 30 \left(\frac{1}{2} \right) \\ F_x &= 15 \text{ N} \end{aligned}$$

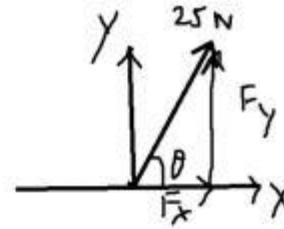
$$\begin{aligned} F_y &= F \sin \theta \\ &= 30 \sin 60^\circ \\ &= 30 \left(\frac{\sqrt{3}}{2} \right) \\ F_y &= 15\sqrt{3} \text{ N} \end{aligned}$$

Example

A force of 25N is acting on the particle as shown in the fig.

Resolved part of force along x-axis is 15N. Find

- (i) The angle that the force makes with x-axis.
- (ii) Resolved part of the force along y-axis.



Sol.

$$F_x = 15 \text{ N}$$

$$F = 25 \text{ N}$$

$$(i) \theta = ?$$

$$(ii) F_y = ?$$

$$F_y = ?$$

$$F_x = F \cos \theta$$

$$\cos \theta = \frac{F_x}{F}$$

$$\theta = \cos^{-1} \left(\frac{15}{25} \right)$$

$$= \cos^{-1} (0.6)$$

$$= 53$$

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{15}{25}\right) \\ &= \cos^{-1}(0.6) \\ &= 53.13^\circ\end{aligned}$$

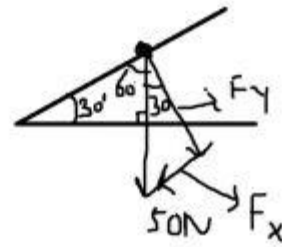
$$\begin{aligned}F^2 &= F_x^2 + F_y^2 \\ 25^2 &= 15^2 + F_y^2 \\ 625 &= 225 + F_y^2 \\ F_y^2 &= 400 \\ F_y &= 20 \text{ N}\end{aligned}$$

Example

A particle of mass 5kg is placed on an inclined plane as shown in the fig. Find the resolved parts of the weight of the particle in the direction parallel and perpendicular to the plane.

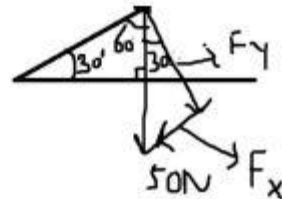
Sol.:-

$$\begin{aligned} m &= 5 \text{ kg} \\ W &= mg \\ &= 5(10) \\ &= 50 \text{ N} \end{aligned}$$



Sol.:-

$$\begin{aligned} m &= 5 \text{ kg} \\ W &= mg \\ &= 5(10) \\ &= 50 \text{ N} \\ F_x &= mg \sin 30^\circ \\ &= 5(10) \sin 30^\circ \\ &= 50 \sin 30^\circ \\ &= 50 \left(\frac{1}{2} \right) \\ &= 25 \text{ N} \end{aligned}$$



[..

$$\begin{aligned} F_x &= mg \sin 30^\circ \\ &= 5(10) \sin 30^\circ \\ &= 50 \sin 30^\circ \\ &= 50 \left(\frac{1}{2} \right) \\ &= 25 \text{ N} \end{aligned}$$

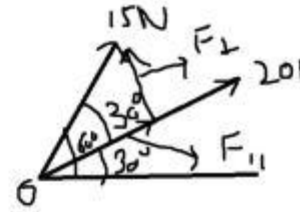
$$\begin{aligned} F_y &= F \cos 30^\circ = mg \cos 30^\circ \\ &= 50 \left(\frac{\sqrt{3}}{2} \right) \\ &= 25 \sqrt{3} \text{ N} \end{aligned}$$

Example

A force of 15N and 20N are acting on the point as shown in the fig. Find the resolved component of the force of 15N in the direction of parallel and perpendicular to the force of 20N.

Sol.

$$\begin{aligned}
 F_{||} &= F \cos \theta \\
 &= 15 \cos 30 \\
 &= 15 \times \frac{\sqrt{3}}{2} \\
 &= 7.5\sqrt{3} \text{ N}
 \end{aligned}$$

Sol.

$$\begin{aligned}
 F_{||} &= F \cos \theta \\
 &= 15 \cos 30 \\
 &= 15 \times \frac{\sqrt{3}}{2} \\
 &= 7.5\sqrt{3} \text{ N} \\
 F_{\perp} &= F \sin \theta \\
 &= 15 \times \sin(30^\circ) \\
 &= 15 \left(\frac{1}{2}\right) \\
 &= 7.5 \text{ N}
 \end{aligned}$$



Lecture No 07

Action - Reaction

- Newton's first two laws of motion explain how the motion of a single object changes.
- Newton's **third law** describes something else that happens when one object exerts a **force** on another object.
- According to **Newton's third law of motion**, forces always act in **equal** but **opposite** pairs.

Action - Reaction

- The forces exerted by **two objects** on each other are often called an **action-reaction force** pair.
- Either force can be considered the action force or the reaction force.
- Action and reaction force pairs don't cancel because they act on different objects.

Newton's Third Law

- Newton's Third Law of Motion states: for every action, there is an equal but opposite reaction.
- This means that when you push on a wall, the wall pushes back on you with a force equal in strength to the force you exerted.

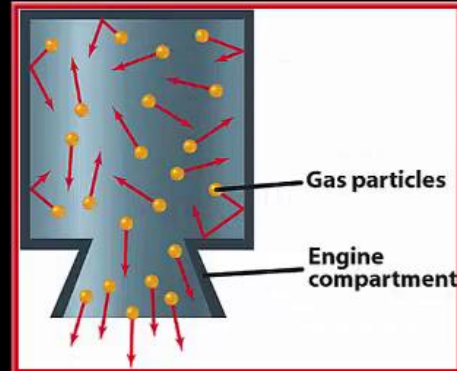
Action - Reaction

- You constantly use action-reaction force pairs as you move about.
- When you jump, you push down on the ground.
- The ground then pushes up on you. It is this upward force that pushes you into the air.



Action - Reaction

- When the rocket fuel is ignited, a hot gas is produced.
- As the gas molecules collide with the inside engine walls, the walls exert a force that pushes them out of the bottom of the engine.

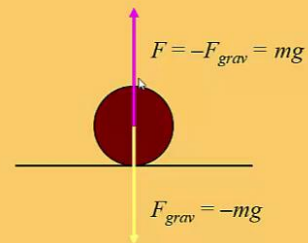


Action - Reaction

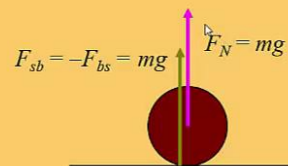
- This downward push is the action force.
- The reaction force is the upward push on the rocket engine by the gas molecules.
- This is the thrust that propels the rocket upward.

Gravity at Rest

- ✿ The force of gravity acts on all objects all the time.
- ✿ If an object is at rest, the law of inertia says that the net force is zero.
- ✿ There must be a force opposite to gravity that cancels it out.



Normal Force



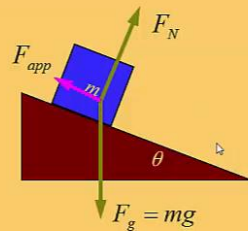
- ✿ The force that opposes gravity for objects on the ground is called the *normal force*.
- ✿ It is perpendicular (normal) to the plane of the ground.
- ✿ The force is a result of the law of reaction.

Normal Force and Weight

- ✿ The normal force pushing up against gravity can be measured.
- ✿ We measure weight with a scale that measures normal force.
- ✿ Weight is a force, not a mass.
- ✿ Pounds measure weight, so force (not mass) can be measured in pounds.

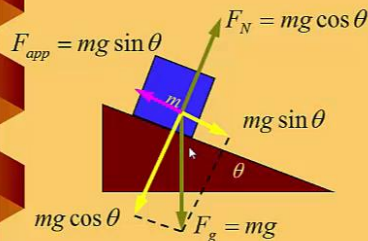


Forces on a Slope



- ✿ Draw the forces acting on the block.
- The force of gravity points down with magnitude $F_g = mg$.
- The normal force F_N points away from the surface of the inclined plane.
- An applied force F_{app} holds the block in place.

Net Force

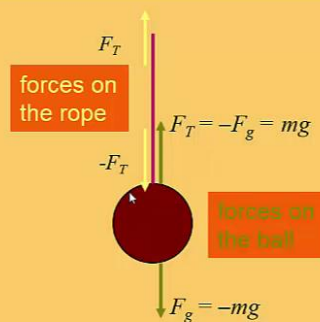


The diagram is the same for constant velocity!

- ✿ The block stays in place on the plane.
 - No net force along the surface
 - No net force away from the surface
- ✿ Find the components of gravity in those directions.
- ✿ Find the unknown forces

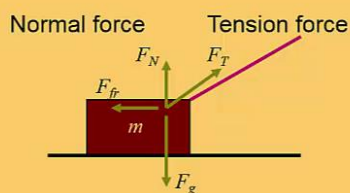
Tension Force

- ✿ A taut rope has a force exerted on it.
- ✿ If the rope is lightweight and flexible the force is uniform over the entire length.
- ✿ This force is called *tension* and points along the rope.
- ✿ Tension acts on both ends.



Tension or Normal Force

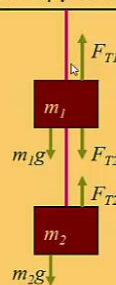
- ✿ Tension and normal forces are different.
 - A pull on an object – tension
 - A push from a surface – normal force
- ✿ Either one or both may be present.



Equilibrium in One Dimension

- ✿ Two weights are hung supported by strings.
 - On the lower block the two forces balance: $F_{T2} = m_2g$
 - On the upper block there are three: $F_{T1} = m_1g + F_{T2}$
 - $F_{T1} = (m_1 + m_2)g$
- ✿ The upper string has more tension than the lower string.

3 forces on the upper block



2 forces on the lower block

Example

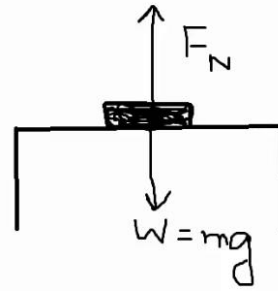
Calculate the normal contact force when a book of mass 3kg is placed on a horizontal table.

Sol

$$m = 3 \text{ kg}$$

$$\begin{aligned} W &= mg \\ &= 3(10) \\ &= 30 \text{ N} \end{aligned}$$

$$\begin{aligned} F_N &= W \\ &= 30 \text{ N} \end{aligned}$$



Example

Two smooth boxes A and B of masses 20kg and 30kg respectively are placed as shown in the diagram. Find the magnitude of the forces acting on each box.

Sol

Box A

$$m_A = 20 \text{ kg}$$

$$W_A = mg$$

$$= 20(10) = 200 \text{ N}$$

$$F_{NA} = W_A = 200 \text{ N}$$

Box B

$$m_B = 30 \text{ kg}$$

$$W_B = mg$$

$$= 30(10) = 300 \text{ N}$$

$$F_{NB} = W_B + F_{NA} = 300 + 200$$

Box B

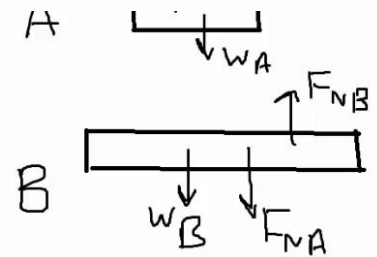
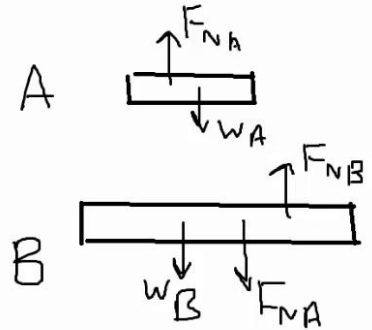
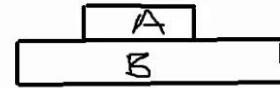
$$m_B = 30 \text{ kg}$$

$$W_B = 30(10)$$

$$= 300 \text{ N}$$

$$F_{NB} = 300 + 200$$

$$= 500 \text{ N}$$



Lecture No 08

Example

Three identical smooth balls each of mass m kg are labelled as A, B and C and they rest in contact with each other in a smooth cylindrical container. Write down, in terms of m and g :

- (i) The magnitude of the force exerted on ball A by ball B
- (ii) The magnitude of the force exerted on ball B by ball C.
- (iii) The magnitude and direction of the resultant force exerted by ball A and C on ball B.

Sol
/

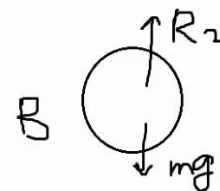
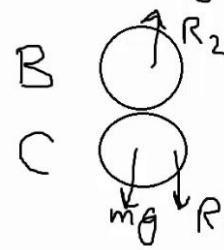
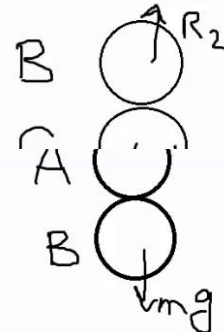
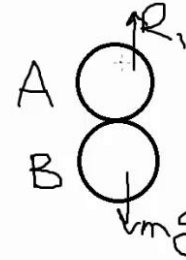
exerted by ball A and C on ball B.

Sol
/

$$(i) \quad R_1 = mg$$

$$(ii) \quad R_2 = mg + R_1 \\ = mg + mg \\ = 2mg$$

$$(iii) \quad R = R_2 - mg \\ = 2mg - mg \\ = mg \text{ act upward}$$



Example

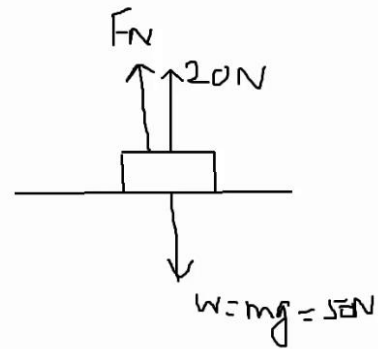
A box of mass 5 kg is placed on a smooth table. A force of 20N is acting on the box as shown in the figure. Find the contact force acting on the box.

Sol:-

$$\begin{aligned} m &= 5 \text{ kg} \\ W &= mg \\ &= 5(10) \\ &= 50 \text{ N} \end{aligned}$$

$$F_N + 20 = 50$$

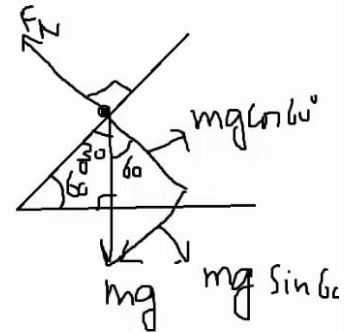
$$\begin{aligned} F_N &= 50 - 20 \\ &= 30 \text{ N} \end{aligned}$$

**Example**

A particle of mass 4kg is placed on an inclined plane whose inclination with the horizontal is 60degrees. The plane is smooth. Find the contact force acting on the particle.

Sol:-

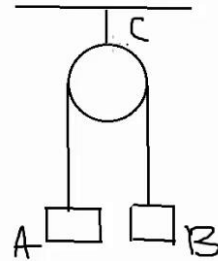
$$\begin{aligned} F_N &= mg \cos 60^\circ \\ &= 4(10) \frac{1}{2} \\ &= 20 \text{ N} \end{aligned}$$



Example

Two particles of mass 2kg each are connected by a light inextensible string passing over a smooth fixed pulley, which is attached to a string C as shown in the fig. the particles are at rest. Find the tension in the string C.

Sol:-



10. Find the tension in the string C.

Sol:-

$$m_A = 2 \text{ kg}$$

$$W_A = mg$$

$$= 2(10)$$

$$= 20 \text{ N}$$

$$W_B = 20 \text{ N}$$

[A] $T_1 = 20 \text{ N}$

B $T_2 = 20 \text{ N}$

At Pulley $T_3 = T_1 + T_2$

$$W_A = mg$$

$$= 2(10)$$

$$= 20 \text{ N}$$

$$W_B = 20 \text{ N}$$

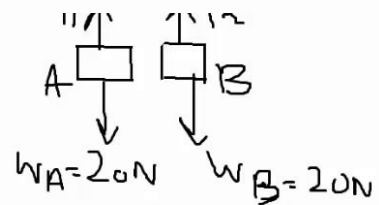
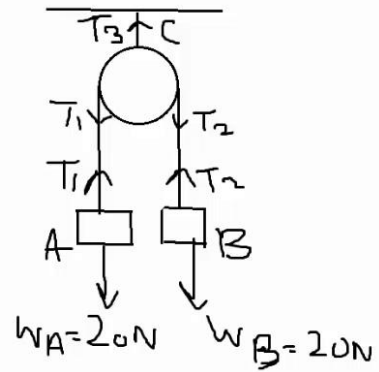
[A] $T_1 = 20 \text{ N}$

B $T_2 = 20 \text{ N}$

At Pulley $T_3 = T_1 + T_2$

$$= 20 + 20$$

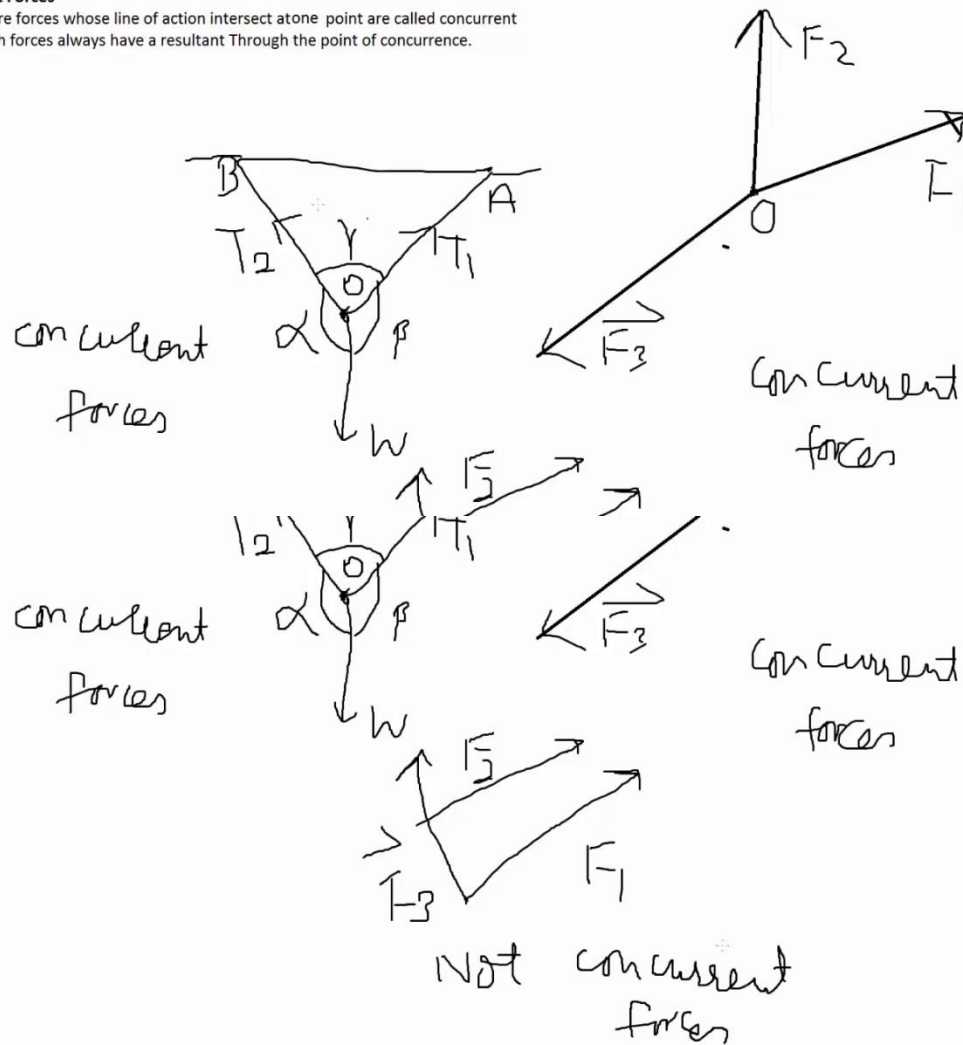
$$= 40 \text{ N}$$



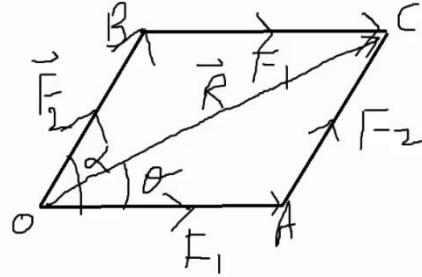
Lecture No 09

Concurrent Forces

Two or more forces whose line of action intersect at one point are called concurrent forces. Such forces always have a resultant through the point of concurrence.



Resultant of two forces acting at a point



$$\vec{R} = \vec{F}_1 + \vec{F}_2 \quad \text{--- (1)}$$

$$\vec{R} \cdot \vec{R} = \vec{R} \cdot (\vec{F}_1 + \vec{F}_2)$$

$$\begin{aligned} \vec{R} &= (\vec{F}_1 + \vec{F}_2) \cdot (\vec{F}_1 + \vec{F}_2) \\ &= \vec{F}_1 \cdot \vec{F}_1 + \vec{F}_2 \cdot \vec{F}_1 + \vec{F}_1 \cdot \vec{F}_2 + \vec{F}_2 \cdot \vec{F}_2 \end{aligned}$$

$$\vec{R} = (\vec{F}_1 + \vec{F}_2) \cdot (\vec{F}_1 + \vec{F}_2)$$

$$= \vec{F}_1 \cdot \vec{F}_1 + \vec{F}_2 \cdot \vec{F}_1 + \vec{F}_1 \cdot \vec{F}_2 + \vec{F}_2 \cdot \vec{F}_2$$

$$\begin{aligned} &= F_1 F_1 \cos 0 + F_2 F_1 \cos \alpha + F_1 F_2 \cos \alpha + F_2 F_2 \cos 0 \\ R^2 &= F_1^2 + 2F_1 F_2 \cos \alpha + F_2^2 \end{aligned}$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \alpha} \quad \text{--- (2)}$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \alpha} \quad \text{--- (2)}$$

$$\vec{F}_1 \cdot \vec{R} = \vec{F}_1 \cdot (\vec{F}_1 + \vec{F}_2)$$

$$F_1 R \cos \theta = F_1^2 + \vec{F}_1 \cdot \vec{F}_2$$

$$F_1 R \cos \theta = F_1^2 + F_1 F_2 \cos \alpha$$

$$R \cos \theta = F_1 + F_2 \cos \alpha \quad \text{--- (3)}$$

$$\vec{F}_1 + \vec{F}_2 \cos \alpha \quad \text{---} \quad \Rightarrow$$

$$|\vec{F}_1 \times \vec{R}| = |\vec{F}_1 \times (\vec{F}_1 + \vec{F}_2)|$$

$$F_1 R \sin \theta = |\vec{F}_1 \times \vec{F}_1 + \vec{F}_1 \times \vec{F}_2|$$

$$F_1 R \sin \theta = |\vec{F}_1 \times \vec{F}_2|$$

$$F_1 R \sin \theta = F_1 F_2 \sin \alpha$$

$$F_1 R \sin \theta = |\vec{F}_1 \times \vec{F}_2|$$

$$F_1 R \sin \theta = F_1 F_2 \sin \alpha$$

$$R \sin \theta = F_2 \sin \alpha \quad \text{---} \quad (4)$$

$$\frac{R \sin \theta}{R \cos \theta} = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\frac{R \sin \theta}{R \cos \theta} = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\theta = \tan^{-1} \left(\frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha} \right)$$

$$\theta = \tan^{-1} \left(\frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha} \right) \quad \text{--- (5)}$$

Case-1

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

R is max $\cos \alpha = 1 \Rightarrow \alpha = 0^\circ$

$$\begin{aligned} \therefore R &= \sqrt{F_1^2 + F_2^2 + 2F_1F_2} \\ &= \sqrt{(F_1 + F_2)^2} \\ R &= F_1 + F_2 \end{aligned}$$

Case-2

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

$\cos \alpha = -1 \Rightarrow R_{\min}$
 $\Rightarrow \alpha = 180^\circ$

$$R_{\min} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2(-1)}$$

$$= \sqrt{F_1^2 + F_2^2 - 2F_1F_2}$$

$\cos \alpha = -1 \Rightarrow R_{\min}$
 $\Rightarrow \alpha = 180^\circ$

$$R_{\min} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2(-1)}$$

$$= \sqrt{(F_1 - F_2)^2}$$

$$= |F_1 - F_2|$$

$R_{\min} = |F_1 - F_2|$
Case-1
 $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$
 $\alpha = 90^\circ$
 $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^\circ}$ $\because \cos 90^\circ = 0$
 $R = \sqrt{F_1^2 + F_2^2}$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$
 $\alpha = 90^\circ$
 $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^\circ}$ $\because \cos 90^\circ = 0$
 $R = \sqrt{F_1^2 + F_2^2}$

$\theta = \tan^{-1}\left(\frac{F_2 \sin 90^\circ}{F_1 + F_2 \cos 90^\circ}\right)$ $\sin 90^\circ = 1$
 $\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right)$

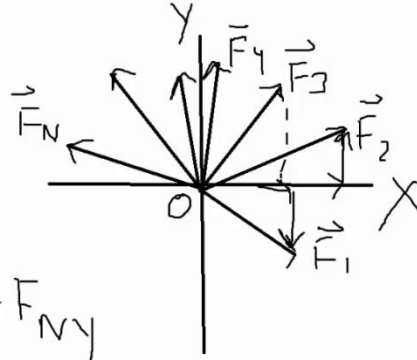
Lecture No 10

Resultant of system of concurrent coplaner forces

$$R_x = F_{1x} + F_{2x} + F_{3x} + \dots + F_{Nx}$$

$$R_y = F_{1y} + F_{2y} + F_{3y} + \dots + F_{Ny}$$

$$R^2 = R_x^2 + R_y^2$$



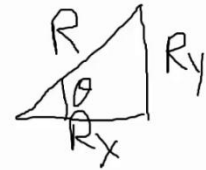
$$R_y = F_{1y} + F_{2y} + F_{3y} + \dots + F_{Ny}$$

$$R^2 = R_x^2 + R_y^2$$

$$R = \sqrt{R_x^2 + R_y^2} \quad \text{--- (1)}$$

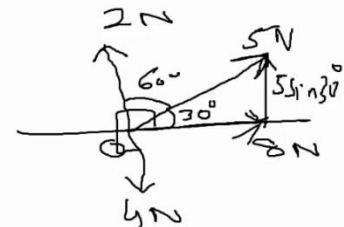
$$\tan \theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right) \quad \text{--- (2)}$$



Example

Find the magnitude and direction of the Resultant of the forces shown in the fig.



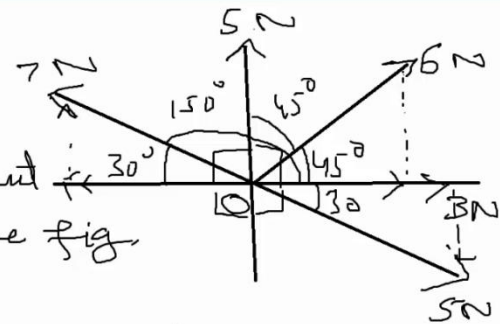
Solution

$$\begin{aligned} X &= 8 \cos 0^\circ + 5 \cos 30^\circ + 2 \cos 90^\circ + 4 \cos 270^\circ \\ &= 8(1) + 5(0.866) + 2(0) + 4(0) \\ &= 8 + 4.33 + 0 + 0 = 12.33 \text{ N} \end{aligned}$$

$$\begin{aligned}
 Y &= 8 \sin 0^\circ + 5 \sin 30^\circ + 2 \sin 90^\circ + 4 \sin 270^\circ \\
 &= 8(0) + 5\left(\frac{1}{2}\right) + 2(1) + 4(-1) \\
 &= 2.5 + 2 - 4 - 0.5 \text{ N} \\
 R &= \sqrt{x^2 + y^2} = \sqrt{12.33^2 + 0.5^2} = \sqrt{152.27} = 12.34 \text{ N} \\
 \theta &= \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{0.5}{12.33}\right) = 2.3^\circ \text{ with } 8 \text{ N force.}
 \end{aligned}$$

Example

Find the magnitude and direction of the resultant of the forces shown in the fig.



Sol.

$$\begin{aligned}
 \rightarrow X &= 3 \cos 0^\circ + 6 \cos 45^\circ + 5 \cos 90^\circ + 7 \cos 150^\circ + 5 \cos 330^\circ \\
 &= 3 + 4.243 + 0 - 6.062 + 4.330 \\
 &= 5.511 \text{ N} \\
 &= 3 + 4.243 + 0 - 6.062 + 4.330 \\
 &= 5.511 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \uparrow Y &= 3 \sin 0^\circ + 6 \sin 45^\circ + 5 \sin 90^\circ + 7 \sin 150^\circ + 5 \sin 330^\circ \\
 &= 0 + 4.243 + 5 + 3.5 - 2.5 \\
 &= 10.243 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 R &= \sqrt{X^2 + Y^2} \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 &= 10.243 \text{ N} \\
 R &= \sqrt{X^2 + Y^2} \\
 &= \sqrt{5.511^2 + 10.243^2} \\
 &= 11.63 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{5.511^2 + 10.243^2} \\
 &= 11.63 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 \theta &= \tan^{-1} \frac{Y}{X} = \tan^{-1} \left(\frac{10.243}{5.511} \right) \\
 &= 61.7^\circ \text{ with } 3 \text{ N force}
 \end{aligned}$$

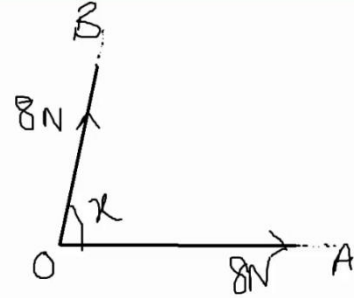
Lecture No 11

Example

Two forces each of magnitude 8N act at a point in the direction OA and OB. The angle between the forces is x . The resultant of the two forces has component 9N in the direction OA. Find

(i) the value of x

(ii) The magnitude of resultant of the two forces.



Sol

$$R_x = 8N + 8\cos x$$

$$9N = 8N + 8\cos x$$

$$8\cos x = 1$$

$$\cos x = \frac{1}{8}$$

$$8\cos x = 1$$

$$\cos x = \frac{1}{8}$$

$$x = 82.8^\circ$$

(ii)

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos x}$$

$$= \sqrt{8^2 + 8^2 + 2 \times 8 \times 8 \times \cos 82.8}$$

$$x = 82.8^\circ$$

(ii)

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos x}$$

$$= \sqrt{8^2 + 8^2 + 2 \times 8 \times 8 \times \cos 82.8}$$

$$= \sqrt{64 + 64 + 128\cos 82.8}$$

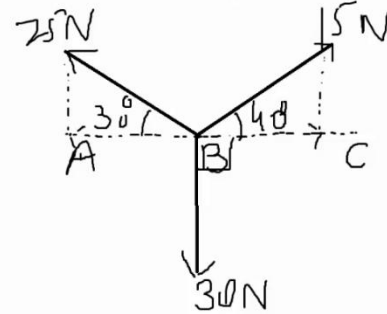
$$= 12N$$

Example

Coplanar forces of magnitudes 15N, 25N and 30N act at a point B on the line ABC in the direction shown in the diagram.

(i) find the magnitude and direction of the resultant force.

(ii) The force of magnitude 15N is now replaced by a force of magnitude FN acting in the same direction. The new resultant force has zero component in the direction BC. Find the value of F and also find the magnitude and direction of the new resultant force.



Sol;

$$R_x = 15 \cos 40^\circ - 25 \cos 30^\circ$$

$$= -10.16$$

$$R_y = 25 \sin 30^\circ + 15 \sin 40^\circ - 30$$

$$R_y = 25 \sin 30^\circ + 15 \sin 40^\circ - 30$$

$$= -7.858$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(-10.16)^2 + (-7.858)^2}$$

$$= 12.8 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

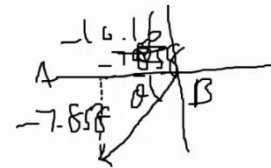
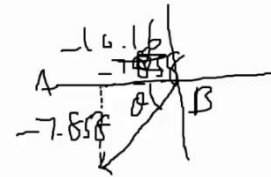
$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(-10.16)^2 + (-7.858)^2}$$

$$= 12.8 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

$$= \tan^{-1} \left(\frac{7.858}{10.16} \right) = 37.7^\circ \text{ with BA downward}$$



downward

(iii)

$$F \cos 40^\circ = 25 \cos 30^\circ$$

$$F = \frac{25 \cos 30^\circ}{\cos 40^\circ}$$

$$= 28.3 \text{ N}$$

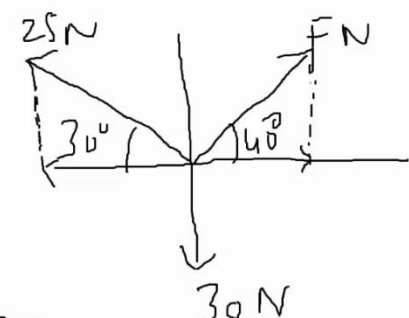
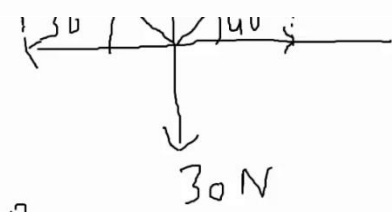
$$R_y = F \sin 40^\circ + 25 \sin 30^\circ - 30$$

$$= \frac{28.3 \sin 40^\circ + 25 \sin 30^\circ - 30}{\cos 40^\circ}$$

$$= 28.3 \text{ N}$$

$$R_y = F \sin 40^\circ + 25 \sin 30^\circ - 30$$

$$= 28.3 \sin 40^\circ + 25 \sin 30^\circ - 30$$

$$= 0.667 \text{ N upwads}$$


Example

Three coplaner forces act at a point. The magnitude of the forces are 20N, 25N and 30N and the direction in which forces act are shown in the diagram, where $\sin A = 0.28$ and $\cos A = 0.96$ and $\sin B = 0.6$ and $\cos B = 0.8$

- Show that the resultant of the three forces has a zero-component in the x-direction.
- Find the magnitude and direction of the resultant of the three forces.
- The force of magnitude 20N is replaced by another force. The effect is that the resultant force is unchanged in the magnitude but reverse in direction. State the magnitude and the direction of the replacement force

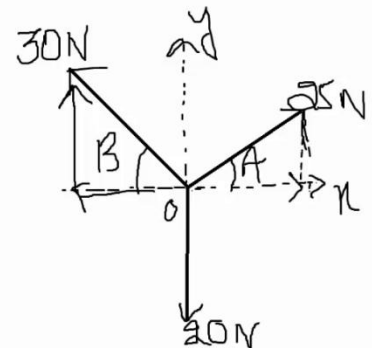
Sol

$$R_x = \sum F_x$$

$$= 25 \cos A - 30 \cos B$$

$$= 25(0.96) - 30(0.8)$$

$$= 24 - 24 = 0$$



$$= 25(0.96) - 30(0.8)$$

$$= 24 - 24 = 0$$

$$(ii) R_y = \sum F_y$$

$$= 25 \sin A + 30 \sin B - 20$$

$$= 25(0.28) + 30(0.6) - 20$$

$$= 5 \text{ N}$$

$$= 25(0.28) + 30(0.6) - 20$$

$$= 5 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{0^2 + (5 \text{ N})^2}$$

$$= \sqrt{25}$$

$$= 5 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

$$= \tan^{-1} \left(\frac{5}{0} \right)$$

$$= 90^\circ$$

$$K = \sqrt{K_x^2 + K_y^2}$$

$$= \sqrt{0^2 + (5 \text{ N})^2}$$

$$= \sqrt{25}$$

$$= 5 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{K_y}{K_x} \right)$$

$$= \tan^{-1} \left(\frac{5}{0} \right)$$

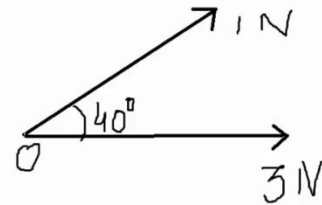
$$= 90^\circ$$

$$(iii) X = 30 \text{ N downward}$$

Lecture No 12

Example

Two forces each of magnitude 1N and 3N act on a particle in the direction as shown in the diagram. Calculate the resultant force on the particle and the angle between the resultant and 3N force.



Sol.

$$R = \sqrt{p^2 + q^2 + 2pq \cos \theta}$$

$$= \sqrt{1^2 + 3^2 + 2(1)(3) \cos 40^\circ}$$

$$= \sqrt{1 + 9 + 6(0.7660)}$$

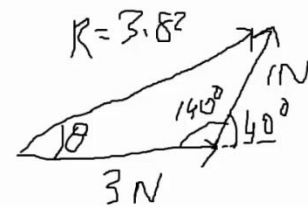
$$= \sqrt{1 + 9 + 6(0.7660)}$$

$$= \sqrt{14.5963}$$

$$= 3.82 \text{ N}$$

Using Sine rule

$$\frac{\sin \theta}{1} = \frac{\sin 140^\circ}{3.82}$$



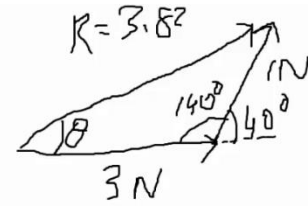
$= 3.82 \text{ N}$
Using Sine rule

$$\frac{\sin \theta}{1} = \frac{\sin 140^\circ}{3.82}$$

$$\sin \theta = 0.1681$$

$$\theta = \sin^{-1}(0.1682)$$

$$= 9.68^\circ = 9.7^\circ \text{ with } 3 \text{ N force}$$

**Example**

Find the magnitude of two forces such that if they act at right angles, their resultant is $\sqrt{10} \text{ N}$ and when they act at an angle of 60° , their resultant is

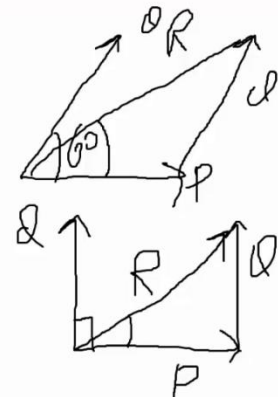
$\sqrt{13} \text{ N}$

Sol: Let the force be p and q

$$R^2 = p^2 + q^2$$

$$(\sqrt{10})^2 = p^2 + q^2$$

$$p^2 + q^2 = 10 \quad \text{--- (1)}$$



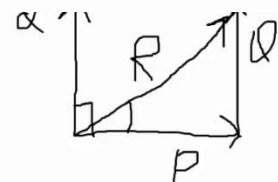
$$(\sqrt{10})^2 = p^2 + q^2$$

$$p^2 + q^2 = 10 \quad \text{--- (1)}$$

$$R^2 = p^2 + q^2 + 2pq \cos \theta$$

$$(\sqrt{13})^2 = p^2 + q^2 + 2pq \cos 60^\circ$$

$$13 = 10 + 2pq \left(\frac{1}{2} \right)$$



$$R^2 = p^2 + q^2 + 2pq \cos \theta$$

$$(\sqrt{13})^2 = p^2 + q^2 + 2pq \cos 60^\circ$$

$$13 = 10 + 2pq \left(\frac{1}{2}\right)$$

$$3 = pq$$

$$pq = 3 \quad (2)$$

$$q = \frac{3}{p} \quad (3)$$

$$q = \frac{3}{p} \quad (3)$$

(1) $\Rightarrow$

$$p^2 + \left(\frac{3}{p}\right)^2 = 10$$

$$p^2 + \frac{9}{p^2} = 10$$

$$p^4 + 9 = 10p^2$$

$$p^4 - 10p^2 + 9 = 0$$

$$p^2 = t \quad (4)$$

$$t^2 - 10t + 9 = 0$$

$$p^4 + 9 = 10p^2$$

$$p^4 - 10p^2 + 9 = 0$$

$$p^2 = t \quad (4)$$

$$t^2 - 10t + 9 = 0$$

$$t^2 - 9t - t + 9 = 0$$

$$t(t-9) - 1(t-9) = 0$$

$$(t-9)(t-1) = 0$$

$$t = 9 \Rightarrow t = 1$$

$$t=1 \Rightarrow p^2=1 \Rightarrow p=1$$

$$t=9 \Rightarrow p^2=9 \Rightarrow p=3$$

$$p=1 \quad p=3$$

$$t=1 \Rightarrow p^2=1 \Rightarrow p=1$$

$$t=9 \Rightarrow p^2=9 \Rightarrow p=3$$

$$p=1 \quad p=3$$

$$\text{using (3)} \\ q = \frac{3}{1} = 3$$

$$q = \frac{3}{3} = 1$$

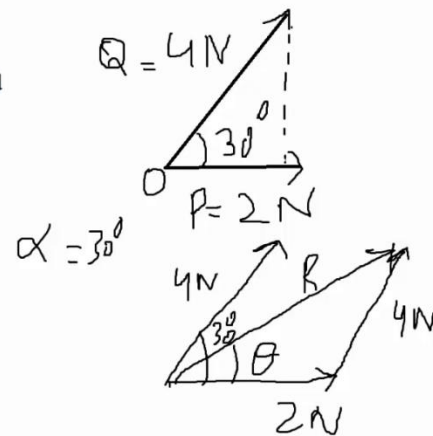
$$\begin{aligned}
 &P^2 + 7 = 0 \quad t = 9 \Rightarrow P = 9 \Rightarrow P = 3 \\
 &P^2 = t \quad (4) \\
 &t^2 - 10t + 9 = 0 \\
 &t^2 - 9t - t + 9 = 0 \\
 &t(t-9) - 1(t-9) = 0 \\
 &(t-9)(t-1) = 0 \\
 &t = 9 \text{ or } t = 1 \\
 &P = 1 \quad P = 3 \\
 &\text{using (3)} \quad q = \frac{3}{1} = 3 \\
 &q = \frac{3}{3} = 1 \\
 &3\text{ N, } 1\text{ N}
 \end{aligned}$$

Example

Two forces each of magnitude 2N and 4N act on a point on the particle in the direction as shown in the diagram. Calculate the angle between the resultant and 2N force.

Sol:-

$$\begin{aligned}
 \tan \theta &= \frac{Q \sin \alpha}{P + Q \cos \alpha} \\
 \tan \theta &= \frac{4 \sin 30^\circ}{2 + 4 \cos 30^\circ} \\
 &=
 \end{aligned}$$



$$\begin{aligned}
 \tan \theta &= \frac{1^2 + 2 \cos \alpha}{4 \sin 30^\circ} \\
 &= \frac{2 + 4 \cos 30^\circ}{4 \left(\frac{1}{2}\right)} \\
 &= \frac{2}{5.46} \\
 &= 0.366
 \end{aligned}$$



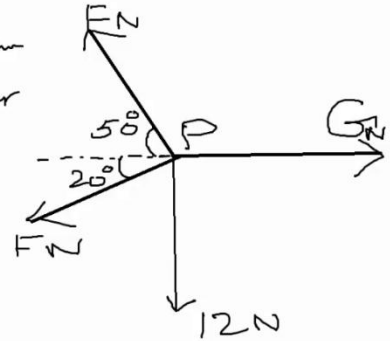
$$\begin{aligned}
 &= \frac{2 + 4 \cos 30^\circ}{4 \left(\frac{1}{2}\right)} \\
 &= \frac{2}{5.46} \\
 &= 0.366
 \end{aligned}$$

2N

$$\begin{aligned}
 \theta &= \tan^{-1}(0.366) \\
 &= 20.1^\circ
 \end{aligned}$$

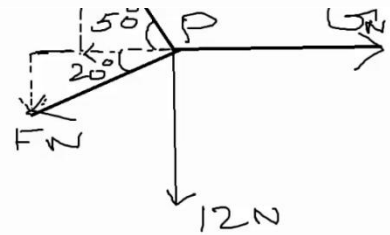
Lecture No 13

Example:- A particle P is in equilibrium on a smooth horizontal table under the action of horizontal forces of the magnitude $F\text{ N}$, $F\text{ N}$ and 12 N acting in the direction shown. Find the value of F and G .



Sol:- $\Sigma X = 0$ and $\Sigma Y = 0$

the action of horizontal forces of the magnitude $F\text{ N}$, $F\text{ N}$ and 12 N acting in the direction shown. Find the value of F and G .



Sol:- $\Sigma X = 0$ and $\Sigma Y = 0$

$$\Sigma Y = 0$$

$$F \sin 50^\circ = F \sin 20^\circ + 12$$

$$X=0 \quad \text{and} \quad Y=0$$

$$Y=0$$

$$F \sin 50^\circ = F \sin 20^\circ + 12$$

$$F \sin 50^\circ - F \sin 20^\circ = 12$$

$$F(\sin 50^\circ - \sin 20^\circ) = 12$$

$$F = \frac{12}{\sin 50^\circ - \sin 20^\circ}$$

$$F = 28.3 \text{ N}$$

$$X=0$$

$$G = F \cos 50^\circ + F \cos 20^\circ$$

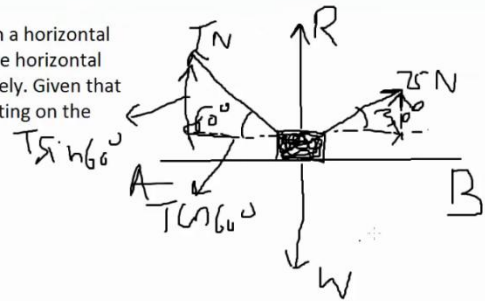
$$= F(\cos 50^\circ + \cos 20^\circ)$$

$$= 28.3 (\cos 50^\circ + \cos 20^\circ)$$

$$= 44.78$$

Ex:

Two light strings are attached to a block of mass 20kg. The block is in equilibrium on a horizontal surface AB with its strings taut. The strings make angle of 60 and 30 degrees with the horizontal on either sides of the block and the tensions in the strings are TN and 75N respectively. Given that the surface is smooth, find the value of T and the magnitude of the contact force acting on the block.



Solution

$$m = 20 \text{ kg}$$

$$W = mg$$

$$= 20(10)$$

$$= 200 \text{ N}$$

$$X=0$$

block.

Solution

$$m = 20 \text{ kg}$$

$$W = mg$$

$$= 20(10)$$

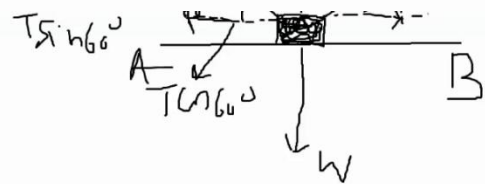
$$= 200 \text{ N}$$

$$X=0$$

$$\Rightarrow T \cos 60^\circ = 75 \cos 30^\circ$$

$$T = \frac{75 \cos 30^\circ}{\cos 60^\circ}$$

$$= 129.9 \text{ N}$$



$$Y=0$$

$$R + T \sin 60^\circ + 75 \sin 30^\circ = 200$$

$$R + 129.9 \sin 60^\circ + 75 \sin 30^\circ = 200$$

$$R = 200 - 129.9 \sin 60^\circ - 75 \sin 30^\circ$$

$$R = 50 \text{ N}$$

Lecture No 14

Example

A Particle P is in equilibrium on a smooth horizontal table under the action of horizontal forces of magnitude 6N, 5N, FN and FN acting in the direction shown. Find the value of α and F.

Sol
→

$$F = 6 \cos \alpha + 5 \cos(90 - \alpha)$$

$$= 6 \cos \alpha + 5 \sin \alpha \quad \text{--- (1)}$$

↓

$$= 6 \cos \alpha + 5 \sin \alpha \quad \text{--- (1)}$$

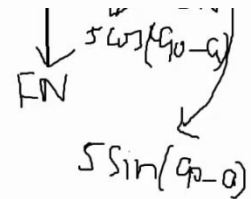
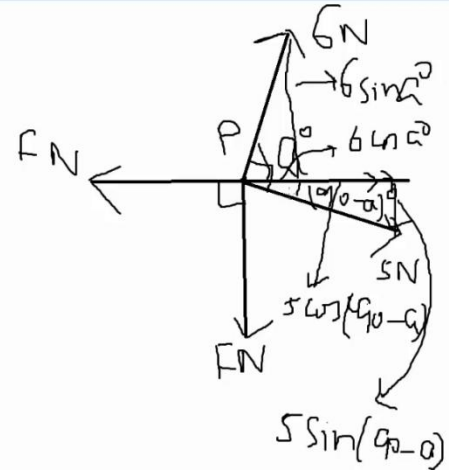
↓

$$F + 5 \sin(90 - \alpha) = 6 \sin \alpha$$

$$F + 5 \cos \alpha = 6 \sin \alpha$$

$$F = 6 \sin \alpha - 5 \cos \alpha \quad \text{--- (2)}$$

$$6 \sin \alpha - 5 \cos \alpha = 6 \cos \alpha + 5 \sin \alpha$$



$$\begin{aligned}
 \checkmark \quad F + 5 \sin(90^\circ - \alpha) &= 6 \sin \alpha^\circ & \Delta \sin(90^\circ - \alpha) \\
 F + 5 \cos \alpha &= 6 \sin \alpha^\circ \\
 F &= 6 \sin \alpha - 5 \cos \alpha \quad \text{--- (2)} \\
 6 \sin \alpha - 5 \cos \alpha &= 6 \cos \alpha + 5 \sin \alpha & \text{using (1) \& (2)} \\
 6 \sin \alpha - 5 \sin \alpha &= 6 \cos \alpha + 5 \cos \alpha \\
 \sin \alpha = 11 \cos \alpha &= \frac{\sin \alpha / \cos \alpha = 11}{\tan \alpha = 11} \\
 \alpha &= \tan^{-1}(11) \\
 &= 84.8^\circ \\
 \text{using (1)} \\
 F &= 6 \cos 84.8^\circ + 5 \sin 84.8^\circ \\
 &= 5.52 \text{ N}
 \end{aligned}$$

Example

Four horizontal forces act a point O and are in equilibrium. the magnitude of the forces are F N, G N 15N, and F N, and the forces act in the direction as shown in the diagram.

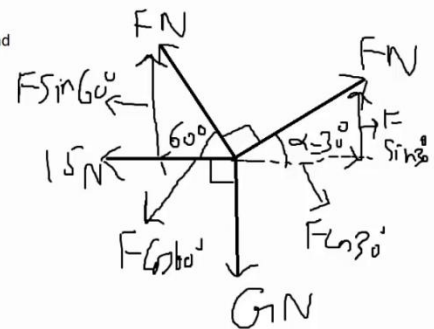
- (i) Show that $F=41.0$ correct to 3 significant figures.
 (ii) Find the value of G.

Solution

$$\alpha + 60^\circ + 90^\circ = 180^\circ$$

$$\alpha = 30^\circ$$

$$\rightarrow F \cos 30^\circ = F \cos 60^\circ + 15$$



$$\alpha = 30^\circ$$

GIN

$$\rightarrow F \cos 30^\circ = F \cos 60^\circ + 15$$

$$F \cos 30^\circ - F \cos 60^\circ = 15$$

$$F(\cos 30^\circ - \cos 60^\circ) = 15$$

$$F = \frac{15}{\cos 30^\circ - \cos 60^\circ}$$

$$F = 40.983$$

$$F(\cos 30^\circ - \cos 60^\circ) = 15$$

$$F = \frac{15}{\cos 30^\circ - \cos 60^\circ}$$

$$F = 40.983$$

$$F = 41.0 \quad \text{correct to 3 s.f.}$$

(ii) ↓

$$G = F \sin 30^\circ + F \sin 60^\circ$$

$$= 41 \sin 30^\circ + 41 \sin 60^\circ$$

(ii) ↓

$$G = F \sin 30^\circ + F \sin 60^\circ$$

$$= 41 \sin 30^\circ + 41 \sin 60^\circ$$

$$= 55.983$$

$$= 56.0 \text{ N}$$

Example

a small block of weight 12 N is at rest on a smooth plane inclined at 40 degrees to the horizontal. The block is held in equilibrium by a force of magnitude PN. find the value of P when
 (i) the force is parallel to the plane as in fig 1
 (ii) The force is horizontal to the plane as in Fig 2.

Sol

$$P = 12 \sin 40^\circ$$

$$= 7.71 \text{ N}$$

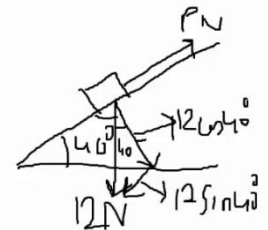
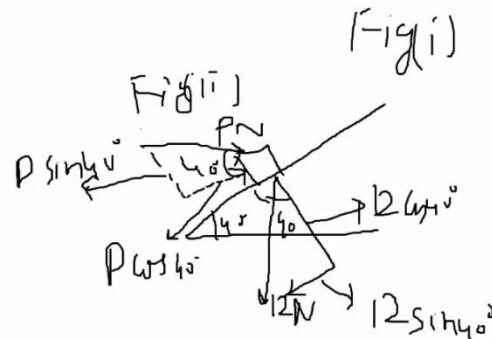
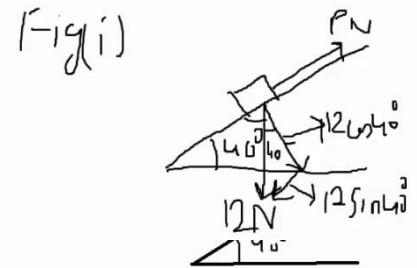
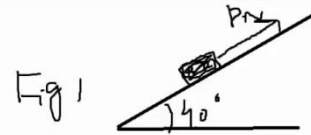
(i) $12 \sin 40^\circ$
 $= 7.71 \text{ N}$

(ii)

$$P \cos 40^\circ = 12 \sin 40^\circ$$

$$P = \frac{12 \sin 40^\circ}{\cos 40^\circ}$$

$$P = 10.1 \text{ N}$$



Lecture No 15

Example

Three coplanar forces of magnitudes F N, 12 N and 15 N are in equilibrium acting at a point P in the direction shown in the diagram. find a and F .

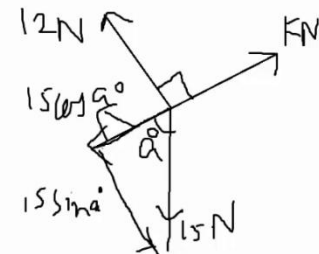
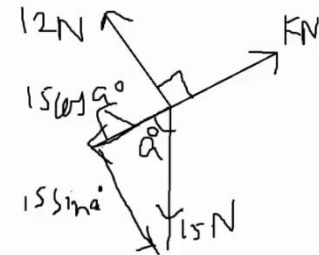
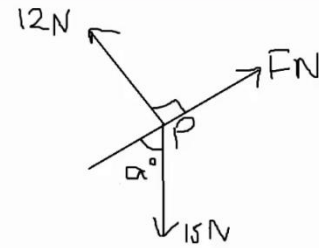
Sol.

$$\begin{aligned} 15 \sin a &= 12 \\ \sin a &= \frac{12}{15} \\ a &= \sin^{-1}\left(\frac{12}{15}\right) \\ &= 53.1^\circ \end{aligned}$$

Sol.

$$\begin{aligned} 15 \sin a &= 12 \\ \sin a &= \frac{12}{15} \\ a &= \sin^{-1}\left(\frac{12}{15}\right) \\ &= 53.1^\circ \end{aligned}$$

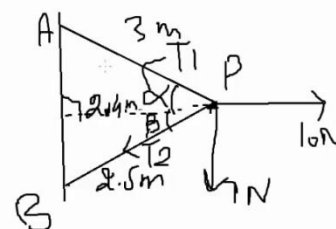
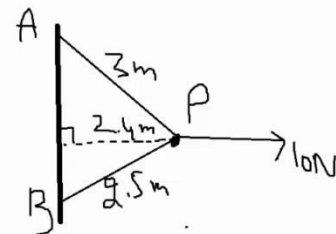
$$\begin{aligned} F &= 15 \cos a^\circ \\ &= 15 \times \cos 53.1^\circ \\ &= 9 \text{ N} \end{aligned}$$

**Example**

A and B are fixed points of a vertical wall with A vertically above B. A particle P of mass 0.7 kg is attached to A by a light inextensible string of length 3 m. P is also attached to B by a light inextensible string of length 2.5 m. P is maintained in equilibrium at a distance of 2.4 m from the wall by a horizontal force of magnitude 10 N acting at P. Both strings are taut and the 10 N force acts in the plane APB which is perpendicular to the wall. Find the tensions in the strings.

Sol.

$$\begin{aligned} m &= 0.7 \text{ kg} \\ w &= mg \\ &= 0.7 \times 10 = 7 \text{ N} \end{aligned}$$



$$w = mg \quad \therefore d$$

$$\Delta AQB \quad = 0.7 \times 10 = 7 \text{ N}$$

$$AQ^2 = AP^2 - BP^2$$

$$AQ = \sqrt{3^2 - 2.4^2} = 1.8 \text{ m}$$

$$\Delta AQB \quad \cos \alpha = \frac{AQ}{AP} = \frac{1.8}{3} = \frac{24}{30} = \frac{4}{5}, \quad \sin \alpha = \frac{BP}{AP} = \frac{2.4}{3} = \frac{18}{30} = \frac{3}{5}$$

$$\Delta QPB \quad QB^2 = PB^2 - QP^2$$

$$QB = \sqrt{2.5^2 - 2.4^2} = 0.7$$

$$\Delta QPB \quad \cos \beta = \frac{QB}{PB} = \frac{0.7}{2.5}$$

$$\sin \beta = \frac{QP}{PB} = \frac{2.4}{2.5}$$

$$\cos \beta = \frac{0.7}{2.5} = \frac{7}{25}$$

$$\sin \beta = \frac{2.4}{2.5} = \frac{24}{25}$$

$$\sin \beta = \frac{2.4}{2.5} = \frac{24}{25}$$

$$\rightarrow T_1 \cos \alpha + T_2 \cos \beta = 10 \text{ N}$$

$$T_1 \times \frac{4}{5} + T_2 \times \frac{24}{25} = 10$$

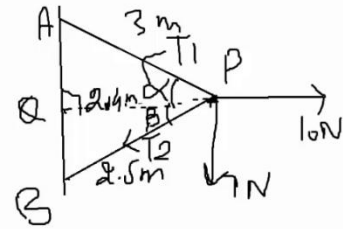
$$T_1 = \frac{5}{4} \left(10 - \frac{24}{25} T_2 \right)$$

$$T_1 \times \frac{4}{5} + T_2 \times \frac{24}{25} = 10$$

$$T_1 = \frac{5}{4} \left(10 - \frac{24}{25} T_2 \right) \quad \text{--- (1)}$$

$$\downarrow T_1 \sin \alpha = T_2 \sin \beta + 7$$

$$T_1 \times \frac{3}{5} = T_2 \times \frac{24}{25} + 7$$



$$T_1 = \frac{5}{3} \left(\frac{7T_2}{25} + 7 \right) \quad \text{--- (2)}$$

using (1) & (2)

$$\frac{7}{4} \left(10 - \frac{24}{25} T_2 \right) = \frac{5}{3} \left(\frac{7T_2}{25} + 7 \right)$$

$$3 \left(10 - \frac{24}{25} T_2 \right) = 4 \left(\frac{7T_2}{25} + 7 \right)$$

$$3 \left(10 - \frac{24}{25} T_2 \right) = 4 \left(\frac{7T_2}{25} + 7 \right)$$

$$0.8 T_2 = 0.4$$

$$T_2 = 0.5 \text{ N}$$

$$T_1 = \frac{5}{3} \left(\frac{7(0.5)}{25} + 7 \right)$$

$$= 11.9 \text{ N}$$

Lecture No 16

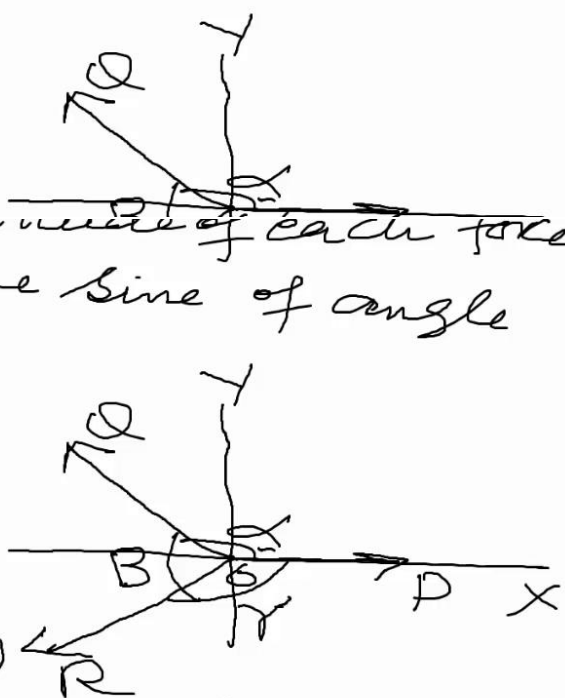
Lami's theorem

If a particle is in equilibrium under the action of three concurrent forces, then the magnitude of each force is proportional to the sine of angle between other two.

Proof ..

is proportional to the sine of angle between other two.

Proof ..

$$X = P \cos \alpha + Q \cos \alpha + R \cos (\alpha + \beta)$$


$$X = P(1) + Q \cos \alpha + R \cos \gamma$$

$$X = P + Q \cos \alpha + R \cos \gamma$$

$$\alpha + \beta + \gamma = 2\pi$$

$$\alpha + \beta = 2\pi - \gamma$$

$$\cos (\alpha + \beta) = \cos (2\pi - \gamma)$$

$$= \cos \gamma$$

$$\begin{aligned}
 X &= P(1) + Q \cos \alpha + R \cos \gamma \\
 X &= P + Q \cos \alpha + R \cos \gamma
 \end{aligned}
 \quad
 \begin{aligned}
 \alpha + \beta + \gamma &= 2\pi \\
 \alpha + \beta &= 2\pi - \gamma \\
 \cos(\alpha + \beta) &= \cos(2\pi - \gamma) \\
 &= \cos \gamma
 \end{aligned}$$

$$\begin{aligned}
 Y &= P \sin 0^\circ + Q \sin \alpha + R \sin(\alpha + \beta) \\
 &= P(0) + Q \sin \alpha + R \sin(\alpha + \beta)
 \end{aligned}$$

$$\begin{aligned}
 \alpha + \beta + \gamma &= 2\pi \\
 \alpha + \beta &= 2\pi - \gamma \\
 \sin(\alpha + \beta) &= \sin(2\pi - \gamma) \\
 Y &= Q \sin \alpha - R \sin \gamma = -\sin \gamma
 \end{aligned}$$

$$X = 0, \quad Y = 0$$

$$X = 0$$

$$\Rightarrow P + Q \cos \alpha + R \cos \gamma = 0 \quad (1)$$

$$Y = 0$$

$$\Rightarrow Q \sin \alpha - R \sin \gamma = 0$$

$$Q \sin \alpha = R \sin \gamma$$

$$\begin{aligned}
 (2) \Rightarrow \quad \frac{R}{\sin \alpha} &= \frac{Q}{\sin \gamma} \quad (2) \\
 Q &= R \sin \gamma
 \end{aligned}$$

$$(2) \Rightarrow \frac{R}{\sin \alpha} = \frac{Q}{\sin r} \quad (2)$$

$$Q = \frac{R \sin r}{\sin \alpha} \quad (3)$$

using 3) in (1)

$$P + R \frac{\sin r}{\sin \alpha} \cos \alpha + R \cos r = 0$$

$$P + R \left(\frac{\sin r}{\sin \alpha} \cos \alpha + \cos r \right) = 0$$

$$P + R \left(\frac{\sin r \cos \alpha + \cos r \sin \alpha}{\sin \alpha} \right) = 0$$

$$P + R \frac{\sin r}{\sin \alpha} \cos \alpha + R \cos r = 0$$

$$P + R \left(\frac{\sin r}{\sin \alpha} \cos \alpha + \cos r \right) = 0$$

$$P + R \left(\frac{\sin r \cos \alpha + \cos r \sin \alpha}{\sin \alpha} \right) = 0$$

$$P + R \frac{\sin (\alpha + r)}{\sin \alpha} = 0$$

$$P + R \frac{(-\sin \theta)}{\sin \alpha} = 0$$

$$1 + R \frac{\sin(\alpha + \beta)}{\sin \alpha} = 0$$

$$P + R \frac{(-\sin \beta)}{\sin \alpha} = 0$$

$$P = \frac{R \sin \beta}{\sin \alpha}$$

$$\frac{P}{\sin \beta} = \frac{R}{\sin \alpha} \quad \text{--- (4)}$$

comb. eq. (2) & (4)

$$\frac{P}{\sin \beta} = \frac{Q}{\sin r} = \frac{R}{\sin \alpha}$$

Example

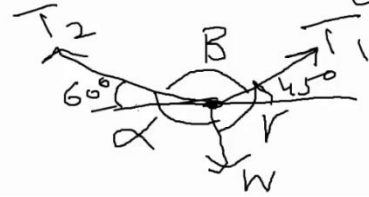
A particle of mass 15 Kg is suspended by two strings. If the angles made by string are 45° and 60° with the horizontal. Find the Tensions in the string.

Sol

$$60^\circ + \beta + 45^\circ = 180^\circ$$

$$\beta = 180^\circ - 60^\circ - 45^\circ$$

$$= 75^\circ$$



$$\beta = 180^\circ - 60^\circ - 45^\circ$$

$$= 75^\circ$$

$$\alpha = 60^\circ + 90^\circ$$

$$= 150^\circ$$

$$\gamma = 90^\circ + 45^\circ$$

$$\gamma = 135^\circ$$



$$\frac{T_1}{\sin \alpha} = \frac{T_2}{\sin \gamma} = \frac{W}{\sin \beta}$$

$$W = mg$$

$$= 15 \times 10$$

$$= 150 \text{ N}$$

$$\frac{T_1}{\sin \alpha} = \frac{T_2}{\sin \gamma} = \frac{W}{\sin \beta}$$

$$= 15 \times 10$$

$$= 150 \text{ N}$$

$$\frac{T_1}{\sin \alpha} = \frac{W}{\sin \beta}$$

$$\frac{T_1}{\sin 150^\circ} = \frac{150}{\sin 75^\circ}$$

$$T_1 = \frac{150 \sin 150^\circ}{\sin 75^\circ} = \frac{150 \times 0.5}{0.9659} = 77.7 \text{ N}$$

$$T_1 = \frac{150 \sin 15^\circ}{\sin 75^\circ} = \frac{150 \times 0.25}{0.9659} = 77.7 \text{ N}$$

$$\frac{T_2}{\sin \alpha} = \frac{W}{\sin \beta}$$

$$\frac{T_2}{\sin 135^\circ} = \frac{150}{\sin 75^\circ}$$

$$T_2 = \frac{150 \times \sin 135^\circ}{\sin 75^\circ}$$

$$\begin{aligned} T_2 &= \frac{150 \times \sin 135^\circ}{\sin 75^\circ} \\ &= \frac{150 \times 0.707}{0.9659} \\ &= 109.8 \text{ N} \end{aligned}$$

$$T_1 = 77.7 \text{ N}$$

$$T_2 = 109.8 \text{ N}$$

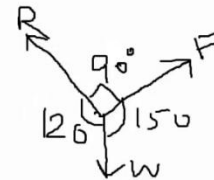
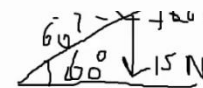
Lecture No 17

Example A block of mass 1.5 kg rests in Equilibrium in a rough inclined plane at 60° to the horizontal. Find the magnitude of the frictional contact force and normal contact force.

Sol: $M = 1.5\text{ kg}$
 $W = mg$
 $= 1.5 \times 10 = 15\text{ N}$
 $W = mg$
 $= 1.5 \times 10 = 15\text{ N}$

$$\frac{F}{\sin 120^\circ} = \frac{R}{\sin 150^\circ} = \frac{15}{\sin 90^\circ}$$

$$\frac{F}{\sin 120^\circ} = \frac{15}{\sin 90^\circ}$$



$$\frac{1}{\sin 120^\circ} = \frac{15}{\sin 90^\circ}$$

$$F = \frac{15}{1} \times \sin 120^\circ$$

$$= 15 \times 0.866$$

$$= 13 \text{ N}$$

$$\frac{R}{\sin 150^\circ} = \frac{15}{\sin 90^\circ}$$

$$= 13 \text{ N}$$

$$\frac{R}{\sin 150^\circ} = \frac{15}{\sin 90^\circ}$$

$$R = \frac{15 \times \sin 150^\circ}{\sin 90^\circ}$$

$$= \frac{15 \times 0.5}{1}$$

$$= 7.5 \text{ N}$$

Example

A particle P of mass 2.1 kg is attached to one end of each of two light inextensible strings. The other ends of the string are attached to points A and B which are at some horizontal level. P hangs in equilibrium at a point 40 cm below the level of A and B, and the strings PA and PB have lengths 50 cm and 104 cm respectively as shown in the diagram. Show that the tension in the string PA is 20 N, and find the tension in the string PB.

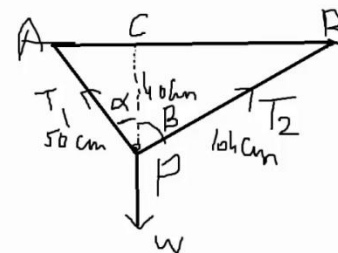
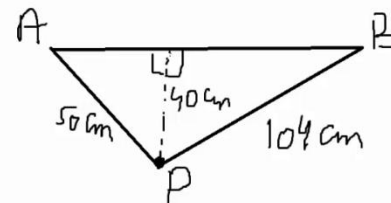
Sol

T_1 = Tension in PA

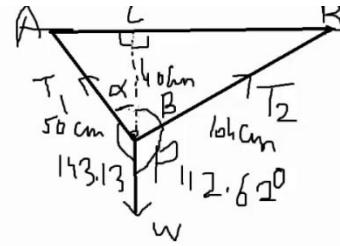
T_2 = Tension in PB

$m = 2.1 \text{ kg}$

$W = mg = 2.1(10) = 21 \text{ N}$



$$\begin{aligned}
 T_1 &= \text{Tension in PA} \\
 T_2 &= \text{Tension in PB} \\
 m &= 2.1 \text{ kg} \\
 W &= mg = 2.1(10) = 21 \text{ N}
 \end{aligned}$$



$$\Delta ACP \quad \cos \alpha = \frac{PC}{PA} = \frac{40}{50} \Rightarrow \alpha = 36.87^\circ$$

$$\Delta BCP \quad \cos B = \frac{PC}{PB} = \frac{40}{104} \Rightarrow B = 67.38^\circ$$

$$\alpha + B = 36.87^\circ + 67.38^\circ = 104.25^\circ$$

$$PB = 104 \Rightarrow B = 67.38^\circ$$

$$\alpha + B = 36.87^\circ + 67.38^\circ = 104.25^\circ$$

By Lami Th.

$$\frac{T_1}{\sin 112.62^\circ} = \frac{T_2}{\sin 143.13^\circ} = \frac{W}{\sin 104.25^\circ}$$

By Lami Th.

$$\frac{T_1}{\sin 112.62^\circ} = \frac{T_2}{\sin 143.13^\circ} = \frac{W}{\sin 104.25^\circ}$$

$$\frac{T_1}{\sin 112.62^\circ} = \frac{21}{\sin 104.25^\circ} \Rightarrow T_1 = \frac{21 \times \sin 112.62^\circ}{\sin 104.25^\circ}$$

$$T_1 = 20 \text{ N}$$

$$\frac{T_1}{\sin 112.62^\circ} = \frac{21}{\sin 104.25^\circ} \Rightarrow T_1 = \frac{21 \times \sin 112.62^\circ}{\sin 104.25^\circ}$$

$$T_1 = 20 \text{ N}$$

$$\frac{T_2}{\sin 143.13^\circ} = \frac{21}{\sin 104.25^\circ} \Rightarrow T_2 = \frac{21}{\sin 104.25^\circ} \times \sin 143.13^\circ$$

$$T_2 = 13 \text{ N}$$

Lecture No 18

Example

Three coplanar forces P N, Q N and 12 N are acting on a particle and are in equilibrium as shown in the figure. Find the magnitudes of the forces P and Q .

Sol.:-

$$\alpha + 90^\circ + 150^\circ = 360^\circ$$

$$\alpha + 240^\circ = 360^\circ$$

$$\alpha = 360 - 240$$

$$= 120^\circ$$

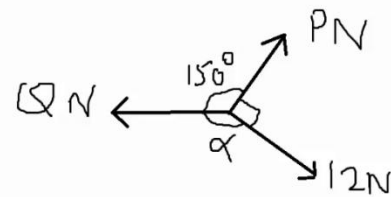
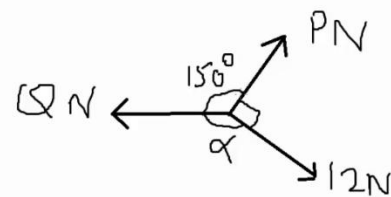
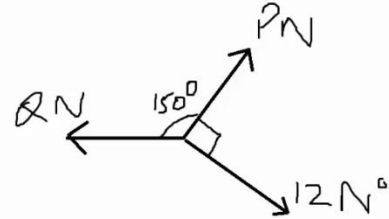
$$\alpha = 360 - 240$$

$$= 120^\circ$$

$$\frac{P}{\sin 120^\circ} = \frac{Q}{\sin 90^\circ} = \frac{12}{\sin 150^\circ}$$

$$\frac{P}{\sin 120^\circ} = \frac{12}{\sin 150^\circ} \Rightarrow P = \frac{\sin 120^\circ \times 12}{\sin 150^\circ} \Rightarrow P = 20.8 \text{ N}$$

$$\frac{Q}{\sin 90^\circ} = \frac{12}{\sin 150^\circ} \Rightarrow Q = \frac{12 \sin 90^\circ}{\sin 150^\circ} \Rightarrow Q = 24 \text{ N}$$



Example

Three coplanar forces are acting at a point O as shown in the fig. The magnitudes of the forces are 6N, 8N and 10N respectively. The angle between the 6N force and the 8N force is 90 degrees. The forces are in equilibrium. Find the other angles between the forces.

Sol

$$\frac{\sin \alpha}{8} = \frac{\sin \beta}{6} = \frac{\sin 90^\circ}{10}$$

$$\frac{\sin \alpha}{8} = \frac{\sin 90^\circ}{10} \Rightarrow \sin \alpha = \frac{8}{10} = 0.8$$

$$\alpha = \sin^{-1} \left(\frac{8}{10} \right) = 53.13^\circ$$

$$\alpha = 180 - 53.13^\circ = 126.87^\circ$$

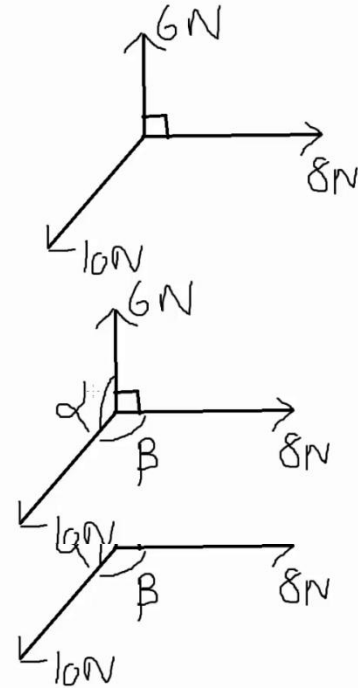
$$\beta + \alpha + 90^\circ = 360^\circ$$

$$\beta + 126.87^\circ + 90^\circ = 360^\circ$$

$$\beta = 360^\circ - 90^\circ - 126.87^\circ$$

$$= 270 - 126.87$$

$$\beta = 143.13^\circ$$



Lecture No 19

Example

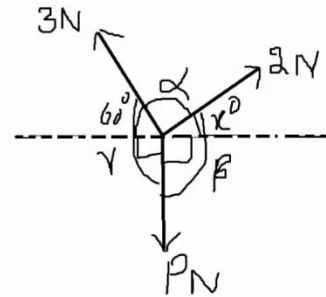
The three concurrent coplanar forces shown in the diagram have magnitudes 3N, 2N and PN. Given that the three forces are in equilibrium, find the value of angle x and P.

Sol

$$\gamma = 60^\circ + 90^\circ = 150^\circ$$

$$\frac{P}{\sin \alpha} = \frac{3}{\sin \beta} = \frac{2}{\sin 150^\circ}$$

$$\text{So } \frac{3}{\sin \beta} = \frac{2}{\sin 150^\circ}$$

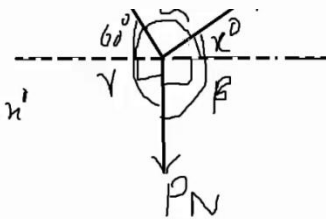


Sol

$$\gamma = 60^\circ + 90^\circ = 150^\circ$$

$$\frac{P}{\sin \alpha} = \frac{3}{\sin \beta} = \frac{2}{\sin 150^\circ}$$

$$\beta = 90^\circ + \alpha$$



$$\text{So } \frac{3}{\sin \beta} = \frac{2}{\sin 150^\circ} \Rightarrow 3 \sin 150^\circ = 2 \sin \beta$$

$$\sin \beta = \frac{3}{2 \times 2} = \frac{3}{4} \Rightarrow \beta = \sin^{-1}\left(\frac{3}{4}\right) = 48.6$$

$$\therefore \beta \text{ is obtuse} \quad \therefore \beta = 180 - 48.6 = 131.4^\circ$$

is obtuse $\therefore \beta = 180 - 48.6 = 131.4^\circ$
 $\gamma = \beta - 90^\circ = 131.4^\circ - 90^\circ = 41.4^\circ \Rightarrow \gamma = 41.4^\circ$
 $\alpha + \beta + \gamma = 360^\circ$
 $\alpha + 131.4^\circ + 150^\circ = 360^\circ \Rightarrow \alpha = 78.6^\circ$
 $\frac{P}{\sin \alpha} = \frac{2}{\sin 150^\circ} \Rightarrow \frac{P}{\sin 78.6^\circ} = \frac{2}{\sin 150^\circ}$
 $P = 4 \times \sin 78.6^\circ$
 $P = 3.92 \text{ N}$

Example

The three coplanar forces shown in the diagram are in equilibrium. Find the values of angle x and force P .

Sol

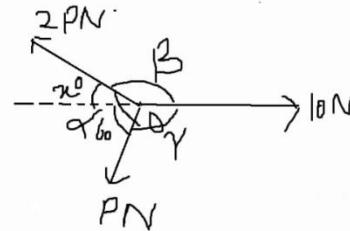
$$\gamma + 60^\circ = 180^\circ \quad \alpha = x + 60^\circ$$

$$\gamma = 120^\circ$$

$$\frac{P}{\sin \beta} = \frac{2P}{\sin 120^\circ} = \frac{10}{\sin \alpha}$$

$$\frac{P}{\sin \beta} = \frac{2P}{\sin 120^\circ}$$

$$\sin \beta = \sin 120^\circ = \frac{\sqrt{3}}{2} \Rightarrow \beta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \beta = 25.7^\circ$$



$$\frac{1}{\sin \beta} = \frac{2P}{\sin 120^\circ} = \frac{10}{\sin \alpha}$$

$$\frac{2P}{\sin \beta} = \frac{2P}{\sin 120^\circ}$$

$$\sin \beta = \frac{\sin 120^\circ}{2} = \frac{\sqrt{3}}{4} \Rightarrow \beta = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right) \Rightarrow \beta = 25.7^\circ$$

$$\beta = 180 - 25.7^\circ = 154.3^\circ$$

$$\gamma = 180 - \beta = 180 - 154.3^\circ \Rightarrow \gamma = 25.7^\circ$$

$$\alpha = \gamma + 60^\circ = 60 + 25.7^\circ = 85.7^\circ$$

$$\alpha = \gamma + 60^\circ = 60 + 25.7^\circ = 85.7^\circ$$

$$\frac{2P}{\sin 120^\circ} = \frac{10}{\sin \alpha}$$

$$2P = \frac{10 \times \sin 120^\circ}{\sin 85.7^\circ}$$

$$P = \frac{10 \times \sin 120^\circ}{2 \times \sin 85.7^\circ} \Rightarrow P = 4.35 \text{ N}$$

Lecture No 20

Proof of Lambda mu theorem

(λ, μ) Theorem If two concurrent forces are represented by $\lambda \vec{OA}$ and $\mu \vec{OB}$, their resultant is given by $(\lambda + \mu) \vec{OC}$, where C divides AB such that $AC : CB = \mu : \lambda$

Proof $\vec{OA} = \vec{OC} + \vec{CA} \quad \text{--- (1)}$

$\vec{OB} = \vec{OC} + \vec{CB} \quad \text{--- (2)}$

(1) $\times \lambda \quad \lambda \vec{OA} = \lambda \vec{OC} + \lambda \vec{CA} \quad \text{--- (3)}$

(2) $\times \mu \quad \mu \vec{OB} = \mu \vec{OC} + \mu \vec{CB} \quad \text{--- (4)}$

(3) + (4) $\lambda \vec{OA} + \mu \vec{OB} = \lambda \vec{OC} + \mu \vec{OC} + \lambda \vec{CA} + \mu \vec{CB} \quad \text{--- (5)}$

$\vec{OA} = \vec{OC} + \vec{CA} \quad \text{--- (1)}$

$\vec{OB} = \vec{OC} + \vec{CB} \quad \text{--- (2)}$

(1) $\times \lambda \quad \lambda \vec{OA} = \lambda \vec{OC} + \lambda \vec{CA} \quad \text{--- (3)}$

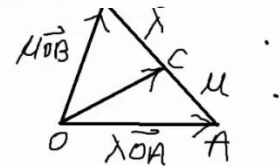
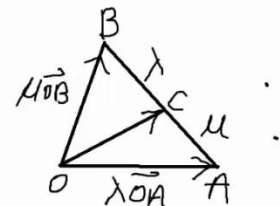
(2) $\times \mu \quad \mu \vec{OB} = \mu \vec{OC} + \mu \vec{CB} \quad \text{--- (4)}$

(3) + (4) $\lambda \vec{OA} + \mu \vec{OB} = \lambda \vec{OC} + \mu \vec{OC} + \lambda \vec{CA} + \mu \vec{CB} \quad \text{--- (5)}$

$AC : CB = \mu : \lambda \quad \lambda \vec{AC} = \mu \vec{CB} \Rightarrow \lambda \vec{CA} + \mu \vec{CB} = 0$

$\lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC} - \lambda \vec{AC} + \mu \vec{CB}$

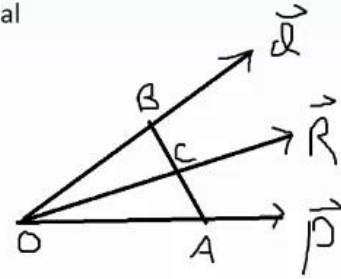
R : $\lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC}$ which is the proof.



Example

Forces **P** and **Q** act at a point and their resultant is **R**. If any transversal cuts three lines of action in the points A, B, C resp. prove that $P/OA + Q/OB = R/OC$

Proof



$$\begin{aligned}\vec{P} &= P \hat{OA} \\ &= P \frac{\vec{OA}}{OA} = \frac{P}{OA} \vec{OA} \quad \text{--- (1)}\end{aligned}$$

$$\vec{Q} = Q \hat{OB} = Q \frac{\vec{OB}}{OB} = \frac{Q}{OB} \vec{OB} \quad \text{--- (2)}$$

$$\vec{R} = R \hat{OC} = R \frac{\vec{OC}}{OC} = \frac{R}{OC} \vec{OC} \quad \text{--- (3)}$$

$$\begin{aligned}\vec{R} &= \vec{P} + \vec{Q} \\ &= \frac{P}{OA} \vec{OA} + \frac{Q}{OB} \vec{OB}\end{aligned}$$

$$\frac{R}{OC} \vec{OC} = \left(\frac{P}{OA} + \frac{Q}{OB} \right) \vec{R} = \left(\frac{P}{OA} + \frac{Q}{OB} \right) \vec{OC}$$

$$\frac{R}{OC} = \frac{P}{OA} + \frac{Q}{OB}$$

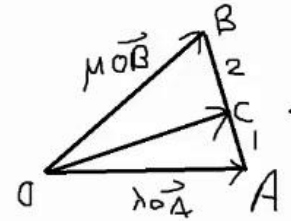
Example

Two concurrent forces are represented by $\lambda \vec{OA}$ and $\mu \vec{OB}$. Find vector $\vec{OC}$ where C divides AB so that $AC:CB=1:2$ and $\vec{OA}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{OB}=\hat{i}+\hat{j}+2\hat{k}$.

Sol:-

$$\mu=1, \lambda=2$$

$$\vec{R} = \lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC}$$

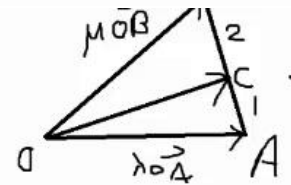


$$\lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC}$$

$$2(\hat{i} + \hat{j} - 2\hat{k}) + 1(\hat{i} + \hat{j} + 2\hat{k}) = (2 + 1) \vec{OC}$$

$$\mu=1, \lambda=2$$

$$\vec{R} = \lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC}$$



$$\lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC}$$

$$2(\hat{i} + \hat{j} - 2\hat{k}) + 1(\hat{i} + \hat{j} + 2\hat{k}) = (2 + 1) \vec{OC}$$

$$2\hat{i} + 2\hat{j} - 4\hat{k} + \hat{i} + \hat{j} + 2\hat{k} = 3 \vec{OC}$$

$$3\hat{i} + 3\hat{j} - 2\hat{k} = 3 \vec{OC}$$

$$\vec{OC} = \hat{i} + \hat{j} - \frac{2}{3}\hat{k}$$

Lecture No 21

Example

Two concurrent forces are represented by $\lambda \vec{OA}$ and $\mu \vec{OB}$. Find vector $\vec{OA}$ where C divides AB so that $AC:CB=3:1$ and $\vec{OB}=2\mathbf{i}+\mathbf{j}-5\mathbf{k}$ and $\vec{OC}=4\mathbf{i}+2\mathbf{j}+\mathbf{k}$. Also find the resultant of these forces.

Sol.

$$\vec{R} = \lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC} \quad (1)$$

$$\mu = 3, \lambda = 1$$

by (1)

$$1\vec{OA} + 3(2\mathbf{i}+\mathbf{j}-5\mathbf{k}) = (3+1)(4\mathbf{i}+2\mathbf{j}+\mathbf{k})$$

$$\vec{OA} + 6\mathbf{i}+3\mathbf{j}-15\mathbf{k} = 4(4\mathbf{i}+2\mathbf{j}+\mathbf{k})$$

$$\mu = 3, \lambda = 1$$

by (1)

$$1\vec{OA} + 3(2\mathbf{i}+\mathbf{j}-5\mathbf{k}) = (3+1)(4\mathbf{i}+2\mathbf{j}+\mathbf{k})$$

$$\vec{OA} + 6\mathbf{i}+3\mathbf{j}-15\mathbf{k} = 4(4\mathbf{i}+2\mathbf{j}+\mathbf{k})$$

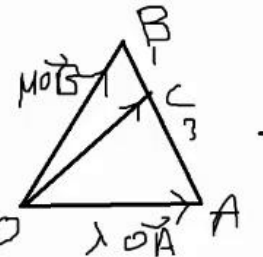
$$\vec{OA} = 16\mathbf{i}+8\mathbf{j}+4\mathbf{k} - 6\mathbf{i}-3\mathbf{j}+15\mathbf{k}$$

$$\text{Now } \vec{OA} = 10\mathbf{i}+5\mathbf{j}+19\mathbf{k}$$

$$\vec{R} = (3+1) \vec{OC}$$

$$= 4(4\mathbf{i}+2\mathbf{j}+\mathbf{k})$$

$$= 16\mathbf{i}+8\mathbf{j}+4\mathbf{k}$$



Example

Two concurrent forces are represented by $\lambda \vec{OA}$ and $\mu \vec{OB}$. Find vector $\vec{OB}$ where C divides AB so that $AC:CB=1:3$ and $\vec{OA}=2\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $\vec{OC}=3\mathbf{i}+3\mathbf{j}+\mathbf{k}$. Also find the resultant of these forces.

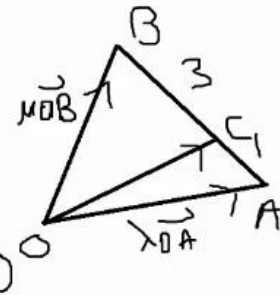
Sol.:-

By $(\lambda + \mu) \vec{OC}$

$$\vec{R} = \lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC} \quad \text{--- (1)}$$

$$\mu = 1, \lambda = 3$$

$$3(2\mathbf{i}+3\mathbf{j}+5\mathbf{k}) + \vec{OB} = (3+1)(3\mathbf{i}+3\mathbf{j}+\mathbf{k})$$



$$\mu = 1, \lambda = 3$$

$$3(2\mathbf{i}+3\mathbf{j}+5\mathbf{k}) + \vec{OB} = (3+1)(3\mathbf{i}+3\mathbf{j}+\mathbf{k})$$

$$6\mathbf{i}+9\mathbf{j}+15\mathbf{k} + \vec{OB} = 4(3\mathbf{i}+3\mathbf{j}+\mathbf{k})$$

$$\begin{aligned} \vec{OB} &= 12\mathbf{i} + 12\mathbf{j} + 4\mathbf{k} - 6\mathbf{i} - 9\mathbf{j} - 15\mathbf{k} \\ &= 6\mathbf{i} + 3\mathbf{j} - 11\mathbf{k} \end{aligned}$$

Now $\vec{R} = (\lambda + \mu) \vec{OC}$

$$= (3+1)(3\mathbf{i}+3\mathbf{j}+\mathbf{k})$$

$$\vec{R} = 12\mathbf{i} + 12\mathbf{j} + 4\mathbf{k}$$

$$\mu = 1, \lambda = 3$$

$$3(2i + 3j + 5k) + \vec{OB} = (3+1)(3i + 3j + k)$$

$$6i + 9j + 15k + \vec{OB} = 4(3i + 3j + k)$$

$$\begin{aligned}\vec{OB} &= 12i + 12j + 4k - 6i - 9j - 15k \\ &= 6i + 3j - 11k\end{aligned}$$

$$\begin{aligned}\text{Now } \vec{R} &= (\lambda + \mu) \vec{OC} \\ &= (3+1)(3i + 3j + k)\end{aligned}$$

$$\vec{R} = 12i + 12j + 4k$$

Example

Two concurrent forces are represented by $\lambda \vec{OA}$ and $\mu \vec{OB}$. Find the magnitude of the resultant force where C divides AB so that $AC:CB=2:3$ and $\vec{OA}=\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $\vec{OC}=\mathbf{4i}-\mathbf{j}+\mathbf{k}$. Also find the resultant of these forces.

Sol.

By (1.4) Th.

$$\vec{R} = \lambda \vec{OA} + \mu \vec{OB} = (\lambda + \mu) \vec{OC}$$

$$\lambda = 3, \quad \mu = 2$$

$$\vec{R} = (\lambda + \mu) \vec{OC}$$

$$R = (\lambda + \mu) OC$$

$$= (3 + 2) (4\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$$= 5 (4\mathbf{i} - \mathbf{j} + \mathbf{k})$$

$$\vec{R} = 20\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$$

$$R = \sqrt{20^2 + 5^2 + 5^2}$$

$$= \sqrt{400 + 25 + 25}$$

$$= \sqrt{450}$$

$$= \sqrt{9 \times 25 \times 2}$$

$$= 3 \times 5 \times \sqrt{2}$$

